Propositional Logic 3: Translation From Natural Language to Propositional Formulas – Problem in Propositional Inferences

Mathematical Logic – First Term 2023-2024

MZI

School of Computing
Telkom University

Soc Tel-U

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Acknowledgements

This slide is compiled using the materials in the following sources:

1. *Discrete Mathematics and Its Applications* (Chapter 1), 8th Edition, 2019, by K. H. Rosen (primary reference).
2. *Discrete Mathematics with Applications* (Chapter 2), 5th Edition, 2018, by S. S. Epp.
3. *Logic in Computer Science: Modelling and Reasoning about Systems* (Chapter 1), 2nd Edition, 2004, by M. Huth and M. Ryan.
4. *Mathematical Logic for Computer Science* (Chapter 2, 3, 4), 2nd Edition, 2000, by M. Ben-Ari.
5. Discrete Mathematics 1 (2012) slides in Fasilkom UI by B. H. Widjaja.
6. Mathematical Logic slides in Telkom University by A. Rakhmatsyah and B. Purnama.

Some figures are excerpted from those sources. This slide is intended for internal academic purpose in SoC Telkom University. No slides are ever free from error nor incapable of being improved. Please convey your comments and corrections (if any) to `<pleasedontspam>@telkomuniversity.ac.id`.

Contents

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3. Wanted: a nurse for a baby about twenty years old.

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4. They are hunting dogs. (What does the speaker mean?)


If your DOG
does a POO
Please put it
in a litter bin.
Please help keep our
open spaces clean.

Sherlock saw the man using binoculars.

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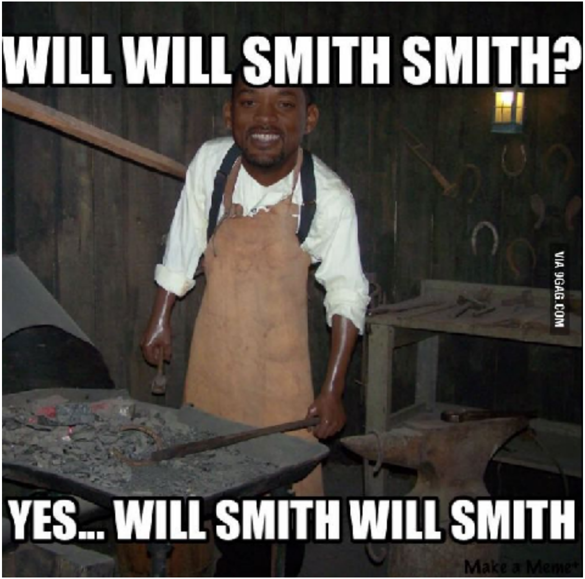
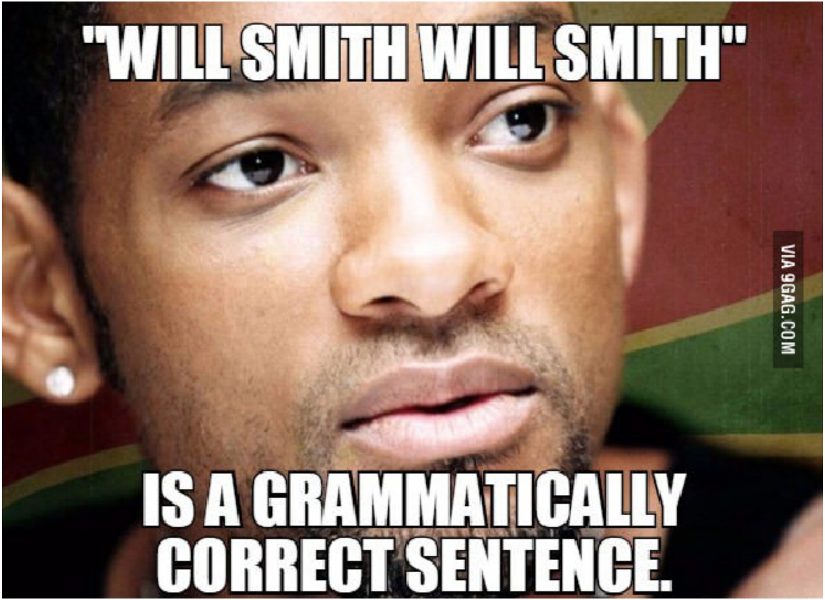

WILL WILL SMITH SMITH?
YES... WILL SMITH WILL SMITH
VIA 9GAG.COM
Make a Meme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Propositional logic and programming languages (such as: Pascal, C, C++, Python, Java) are examples of formal language. This kind of language is suitable for describing and developing software specification due to its logical clarity.

Translation from Natural Language to Propositional Formulas (1)

Exercise

Let p and q be following propositions:

$$p: \text{ "Alex is smart"} \qquad q: \text{ "Alex is cute"}$$

Translate each of the following sentences in propositional formulas:

1. "Alex is smart and cute"
2. "Alex is smart, but he isn't cute"
3. "Either Alex is smart or cute, but not both"
4. "It is not true that Alex is smart or cute.
5. "If Alex is smart, then he isn't cute"

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Solution: (1) $p \wedge q$, (2)

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Translation from Natural Language to Propositional Formulas (2)

Exercise

Express following statements in propositional formulas:

1. "You can vote in the election if you are not under 17 years old, unless you have been married."
2. "You cannot have a driving license if your height is less than $140\,\mathrm{cm}$, unless you use a special car."
3. "If a student does not wear shoes or does not wear shirt, then he/she cannot participate in the exam."

Solution:

For the first sentence, suppose $p :$ "you can vote in the election", $q :$ "you are under 17 years old", and $r :$ "you have been married".

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- $p \to (\neg q \vee r)$ is equivalent to $(q \wedge \neg r) \to \neg p$

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- $p \to (\neg q \vee r)$ is equivalent to $(q \wedge \neg r) \to \neg p$.

For the third sentence, suppose p : "student wears shoes", q : "students wears shirt", and r : "student can participate in the exam".

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Recall that a set/ collection of formulas $\{A_1, A_2, \ldots, A_n\}$ is *consistent* if there exists an interpretation $\mathcal{I}$ such that

$$\mathcal{I}(A_1) = \mathcal{I}(A_2) = \cdots \mathcal{I}(A_n) = \mathrm{T}.$$

Review the following problem.

System's Specifications Consistency Problem

A software engineer is inquired by his manager to develop an information system that complies following specifications:

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System's Specifications Consistency Problem

A software engineer is inquired by his manager to develop an information system that complies following specifications:

1. Whenever the system software is being upgraded, the user cannot access file system;
2. If the user can access file system, then the user can save a new file;
3. If the user cannot save a new file, then the system software is not being upgraded.

Are the above specifications consistent?

System's Specifications Consistency (1)

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- To check the specifications' consistency, we first need to translate each of the specifications to its corresponding propositional formula.
- In order to be consistent, the formulas must not contradictory to one another. This also means that the *conjunction* of the formulas must be true for some interpretation.
- Therefore, if the systems consist of n specification formulas $A_1, A_2, \ldots, A_n$, then there must exist an interpretation $\mathcal{I}$ such that

$$\mathcal{I}(A_1) = \mathcal{I}(A_2) = \cdots = \mathcal{I}(A_n) = \mathrm{T}.$$

To answer the system's specifications consistency problem described previously, we need to translate the system specifications into propositional formulas. Suppose p : "system software is being upgraded", q : "the user can access file system", r : "the user can save a new file".

As a result, the sentences describing the system specification can be written as

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We now determine whether there exist an interpretation $\mathcal{I}$ such that $\mathcal{I}(A_1) = \mathcal{I}(A_2) = \mathcal{I}(A_3) =$

To answer the system's specifications consistency problem described previously, we need to translate the system specifications into propositional formulas. Suppose p : "system software is being upgraded", q : "the user can access file system", r : "the user can save a new file".

As a result, the sentences describing the system specification can be written as

$$\begin{aligned} A_1 &:= p \rightarrow \neg q \\ A_2 &:= q \rightarrow r \\ A_3 &:= \neg r \rightarrow \neg p \end{aligned}$$

We now determine whether there exist an interpretation $\mathcal{I}$ such that $\mathcal{I}(A_1) = \mathcal{I}(A_2) = \mathcal{I}(A_3) = \mathrm{T}$. Observe that, by choosing $\mathcal{I}(p) =$

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Hence, we conclude that the system's specifications are consistent.

Besides the previous method, we can also use truth table to determine the consistency of a system specification. Observe that the formulas of interest are:

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The system specification is consistent if we can find an interpretation $\mathcal{I}$ for each atomic proposition so that $\mathcal{I}(A_1) = \mathcal{I}(A_2) = \mathcal{I}(A_3) = \mathrm{T}$. We have the following truth table:

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p	q	r	$\neg p$	$\neg q$	$\neg r$	$A_1 = p \to \neg q$	$A_2 = q \to r$	$A_3 = \neg r \to \neg p$
1	1	1						

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1	1	1	0	0	0	0	1	1
1	1	0						

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1	1	1	0	0	0	0	1	1
1	1	0	0	0	1	0	0	0
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1	1	1	0	0	0	0	1	1
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1	1	0	0	0	1	0	0	0
1	0	1	0	1	0	1	1	1
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1	0	1	0	1	0	1	1	1
1	0	0	0	1	1	1	1	0
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0	1	1	1	0	0	1	1	1
0	1	0	1	0	1	1	0	1
0	0	1	1	1	0	1	1	1
0	0	0	1	1	1	1	1	1

Since there is at least one interpretation that makes $\mathcal{I}(A_1) = \mathcal{I}(A_2) = \mathcal{I}(A_3) = \mathrm{T}$, then system specification is consistent.

System's Specifications Consistency (2)

Exercise

Check whether the following system specifications are consistent or not:
"The system is in multiuser state if and only if it is operating normally. If the system is operating normally, then the system kernel is functioning. The system kernel is not functioning or the system is in interrupt mode. The system is not in interrupt mode".

Solution:

System's Specifications Consistency (2)

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To translate the specifications , we first define the following atomic propositions:

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Solution:
To translate the specifications , we first define the following atomic propositions: p : "system in multiuser state", q : "system is operating normally", r : "system kernel is functioning", and s : "system is in interrupt mode".
As a result, the specifications can be written as:

$$A_1 :=$$

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$$A_1 := p \leftrightarrow q,\ A_2 :=$$

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Next we now determine whether there exist an interpretation $\mathcal{I}$ such that

$$\mathcal{I}(A_1) = \mathcal{I}(A_2) = \mathcal{I}(A_3) = \mathcal{I}(A_4) = \mathrm{T}$$
$$\mathcal{I}(p \leftrightarrow q) = \mathcal{I}(q \rightarrow r) = \mathcal{I}(\neg r \vee s) = \mathcal{I}(\neg s) = \mathrm{T}$$

Observe that, by choosing $\mathcal{I}(s) =$

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Observe that, by choosing $\mathcal{I}(s) = \mathrm{F}$, $\mathcal{I}(r) =$

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Observe that, by choosing $\mathcal{I}(s) = \mathrm{F}$, $\mathcal{I}(r) = \mathrm{F}$, $\mathcal{I}(q) = \mathrm{F}$, and $\mathcal{I}(p) = \mathrm{F}$, we have

$$\mathcal{I}(A_1) \quad = \quad \mathcal{I}(p \leftrightarrow q) =$$

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Observe that, by choosing $\mathcal{I}(s) = \mathrm{F}$, $\mathcal{I}(r) = \mathrm{F}$, $\mathcal{I}(q) = \mathrm{F}$, and $\mathcal{I}(p) = \mathrm{F}$, we have

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\mathcal{I}(A_4) &= \mathcal{I}(\neg s) =
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Observe that, by choosing $\mathcal{I}(s) = \mathrm{F}$, $\mathcal{I}(r) = \mathrm{F}$, $\mathcal{I}(q) = \mathrm{F}$, and $\mathcal{I}(p) = \mathrm{F}$, we have

$$\begin{aligned} \mathcal{I}(A_1) &= \mathcal{I}(p \leftrightarrow q) = \mathrm{T} \\ \mathcal{I}(A_2) &= \mathcal{I}(q \rightarrow r) = \mathrm{T} \\ \mathcal{I}(A_3) &= \mathcal{I}(\neg r \vee s) = \mathrm{T} \\ \mathcal{I}(A_4) &= \mathcal{I}(\neg s) = \mathrm{T} \end{aligned}$$

Hence, we conclude that the system's specifications are consistent.

We can also use the truth table method to determine the system specification consistency. *However, it should be noted that truth table method is not always efficient*. Observe that our formulas are:

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The system specification is consistent if we can find an interpretation $\mathcal{I}$ for each atomic proposition so that $\mathcal{I}\left(A_1\right) = \mathcal{I}\left(A_2\right) = \mathcal{I}\left(A_3\right) = \mathrm{T}$. We have the following truth table of 16 rows:

We can also use the truth table method to determine the system specification consistency. *However, it should be noted that truth table method is not always efficient*. Observe that our formulas are:

$$A_1 := p \leftrightarrow q,\ A_2 := q \rightarrow r,\ A_3 := \neg r \vee s,\ A_4 := \neg s$$

The system specification is consistent if we can find an interpretation $\mathcal{I}$ for each atomic proposition so that $\mathcal{I}(A_1) = \mathcal{I}(A_2) = \mathcal{I}(A_3) = \mathrm{T}$. We have the following truth table of 16 rows:

p	q	r	s	$\neg r$	$\neg s$	$A_1 = p \leftrightarrow q$	$A_2 = q \rightarrow r$	$A_3 = \neg r \vee s$	$A_4 = \neg s$
1	1	1	1	0	0	1	1	1	0
1	1	1	0	0	1	1	1	0	1
1	1	0	1	1	0	1	0	1	0
1	1	0	0	1	1	1	0	1	1
1	0	1	1	0	0	0	1	1	0
1	0	1	0	0	1	0	1	0	1
1	0	0	1	1	0	0	1	1	0
1	0	0	0	1	1	0	1	1	1

p	q	r	s	$\neg r$	$\neg s$	$A_1 = p \leftrightarrow q$	$A_2 = q \rightarrow r$	$A_3 = \neg r \vee s$	$A_4 = \neg s$
0	1	1	1	0	0	0	1	1	0
0	1	1	0	0	1	0	1	0	1
0	1	0	1	1	0	0	0	1	0
0	1	0	0	1	1	0	0	1	1
0	0	1	1	0	0	1	1	1	0
0	0	1	0	0	1	1	1	0	1
0	0	0	1	1	0	1	1	1	0
0	0	0	0	1	1	1	1	1	1

p	q	r	s	$\neg r$	$\neg s$	$A_1 = p \leftrightarrow q$	$A_2 = q \rightarrow r$	$A_3 = \neg r \vee s$	$A_4 = \neg s$
0	1	1	1	0	0	0	1	1	0
0	1	1	0	0	1	0	1	0	1
0	1	0	1	1	0	0	0	1	0
0	1	0	0	1	1	0	0	1	1
0	0	1	1	0	0	1	1	1	0
0	0	1	0	0	1	1	1	0	1
0	0	0	1	1	0	1	1	1	0
0	0	0	0	1	1	1	1	1	1

Since there is at least one interpretation that makes $\mathcal{I}(A_1) = \mathcal{I}(A_2) = \mathcal{I}(A_3) = \mathcal{I}(A_4) = \mathrm{T}$, then system specification is consistent.

Contents

Logic Puzzles

Formulas' collection consistency can be implemented to answer following problem.

Exercise (*Knights and Knaves*)

Inhabitants of an island can be divided into the knight and the knave. Knights always tell the truth, whereas knaves always tell a lie. In addition, the knights like to help people, while the knaves are notorious for their cannibalism.

One day, you are stranded on this island. Fortunately, you know that an inhabitant of this island is either a knight or a knave. You encounter two people, Pluck and Qluck. Pluck says, "At least one of us is a knave" and Qluck says nothing.

Can you determine who is the knight and/or who is the knave?

Exercise (*The Bank Robbery*)

Five convicts: Abby, Heather, Kevin, Randy, and Vijay, are suspected in a bank robbery. The police did not know exactly who among the five people were involved in the robbery. However, from credible sources, the police learned that the following facts are true:

1. Either Kevin or Heather or both were involved in the robbery.
2. One of Randy or Vijay, but not both, were involved in the robbery.
3. If Abby robbed the bank, so did Randy.
4. Vijay and Kevin were both involved in the robbery, or not at all.
5. If Heather robbed the bank, then so did Abby and Kevin

Is it possible to determine who did rob the bank from the above information alone? If so, who did rob the bank?

Contents

Logical Argument

Logical Argument

A logical argument (or simply an argument) is a finite sequence of propositions.

All but the *final proposition* in the argument are called the *premises* (*assumptions*/*hypotheses*), while the last one is called the *conclusion*.
An argument is *valid*/*sound* if the truth of all its premises implies the truth of its conclusion.

Logical Argument

Logical Argument

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All but the *final proposition* in the argument are called the *premises* (*assumptions*/*hypotheses*), while the last one is called the *conclusion*.
An argument is *valid*/*sound* if the truth of all its premises implies the truth of its conclusion.

From the above definition, an argument with premises $p_1, p_2, \ldots, p_n$ and a conclusion q is valid when we have $(p_1 \wedge p_2 \wedge \cdots \wedge p_n) \Rightarrow q$, or in other words $(p_1 \wedge p_2 \wedge \cdots \wedge p_n) \rightarrow q$ is a tautology.

Elementary Propositional Inference Rules

The elementary rules of inference for propositional logic includes:

1. modus ponens
2. modus tollens
3. double negation introduction
4. double negation elimination
5. hypothetical syllogism
6. disjunctive syllogism
7. addition/ disjunction introduction
8. simplification
9. conjunction
10. resolution

Modus Ponens

Modus Ponens

Let p and q be propositions,

$$\begin{array}{l} p \to q \\ \underline{p \qquad\quad} \\ \therefore q \end{array}$$

Modus Ponens

Modus Ponens

Let p and q be propositions,

$$\begin{array}{l} p \rightarrow q \\ p \\ \hline \therefore q \end{array}$$

Observe that $((p \rightarrow q) \wedge p) \rightarrow q$ is a tautology, hence we have $((p \rightarrow q) \wedge p) \Rightarrow q$.

Example

Modus Ponens

Modus Ponens

Let p and q be propositions,

$$\begin{array}{l} p \to q \\ p \\ \hline \therefore q \end{array}$$

Observe that $((p \to q) \wedge p) \to q$ is a tautology, hence we have $((p \to q) \wedge p) \Rightarrow q$.

Example

If Andre studies in Bandung, then he lives in Indonesia.
Andre studies in Bandung.

Modus Ponens

Modus Ponens

Let p and q be propositions,

$$\begin{array}{l} p \rightarrow q \\ p \\ \hline \therefore q \end{array}$$

Observe that $((p \rightarrow q) \wedge p) \rightarrow q$ is a tautology, hence we have $((p \rightarrow q) \wedge p) \Rightarrow q$.

Example

If Andre studies in Bandung, then he lives in Indonesia.

Andre studies in Bandung.

$\therefore$ Andre lives in Indonesia.

Modus Tollens

Modus Tollens

Let p and q be propositions,

$$\begin{array}{l} p \rightarrow q \\ \neg q \\ \hline \therefore \neg p \end{array}$$

Modus Tollens

Modus Tollens

Let p and q be propositions,

$$\begin{array}{l} p \rightarrow q \\ \neg q \\ \hline \therefore \neg p \end{array}$$

Observe that $((p \rightarrow q) \wedge \neg q) \rightarrow \neg p$ is a tautology, hence we have $((p \rightarrow q) \wedge \neg q) \Rightarrow \neg p$.

Example

Modus Tollens

Modus Tollens

Let p and q be propositions,

$$\begin{array}{l} p \to q \\ \underline{\neg q \quad\quad} \\ \therefore \neg p \end{array}$$

Observe that $((p \to q) \land \neg q) \to \neg p$ is a tautology, hence we have $((p \to q) \land \neg q) \Rightarrow \neg p$.

Example

If Andre studies in Bandung, then he lives in Indonesia.

Andre doesn't live in Indonesia

Modus Tollens

Modus Tollens

Let p and q be propositions,

$$\begin{array}{l} p \rightarrow q \\ \neg q \\ \hline \therefore \neg p \end{array}$$

Observe that $((p \rightarrow q) \wedge \neg q) \rightarrow \neg p$ is a tautology, hence we have $((p \rightarrow q) \wedge \neg q) \Rightarrow \neg p$.

Example

If Andre studies in Bandung, then he lives in Indonesia.

Andre doesn't live in Indonesia

$\therefore$ Andre doesn't study in Bandung.

Double Negation Introduction

Double Negation Introduction

Let p be a proposition,

$$\frac{p}{\therefore \neg\neg p}$$

Double Negation Introduction

Double Negation Introduction

Let p be a proposition,

$$\begin{array}{l} p \\ \hline \therefore \neg\neg p \end{array}$$

Observe that $p \rightarrow \neg\neg p$ is a tautology, hence we have $p \Rightarrow \neg\neg p$.

Example

Double Negation Introduction

Double Negation Introduction

Let p be a proposition,

$$\frac{p}{\therefore \neg\neg p}$$

Observe that $p \rightarrow \neg\neg p$ is a tautology, hence we have $p \Rightarrow \neg\neg p$.

Example

$$\frac{\text{Andre studies in Bandung}}{}$$

Double Negation Introduction

Double Negation Introduction

Let p be a proposition,

$$\frac{p}{\therefore \neg\neg p}$$

Observe that $p \rightarrow \neg\neg p$ is a tautology, hence we have $p \Rightarrow \neg\neg p$.

Example

Andre studies in Bandung

$\therefore$ It is not true that Andre doesn't study in Bandung.

Double Negation Elimination

Double Negation Elimination

Let p be a proposition,

$$\frac{\neg\neg p}{\therefore p}$$

Double Negation Elimination

Double Negation Elimination

Let p be a proposition,

$$\frac{\neg\neg p}{\therefore p}$$

Observe that $\neg\neg p \rightarrow p$ is a tautology, hence we have $\neg\neg p \Rightarrow p$.

Example

Double Negation Elimination

Double Negation Elimination

Let p be a proposition,

$$\frac{\neg\neg p}{\therefore p}$$

Observe that $\neg\neg p \to p$ is a tautology, hence we have $\neg\neg p \Rightarrow p$.

Example

$$\frac{\text{It is not true that Andre doesn't study in Bandung.}}{}$$

Double Negation Elimination

Double Negation Elimination

Let p be a proposition,

$$\frac{\neg\neg p}{\therefore p}$$

Observe that $\neg\neg p \rightarrow p$ is a tautology, hence we have $\neg\neg p \Rightarrow p$.

Example

$$\frac{\text{It is not true that Andre doesn't study in Bandung.}}{\therefore \text{Andre studies in Bandung.}}$$

Hypothetical Syllogism

Hypothetical Syllogism

Let p, q, r be propositions,

$$\begin{array}{c} p \rightarrow q \\ q \rightarrow r \\ \hline \therefore p \rightarrow r \end{array}$$

Hypothetical Syllogism

Hypothetical Syllogism

Let p, q, r be propositions,

$$\begin{array}{c} p \to q \\ \underline{q \to r} \\ \therefore p \to r \end{array}$$

Observe that $((p \to q) \wedge (q \to r)) \to (p \to r)$ is a tautology, hence we have $((p \to q) \wedge (q \to r)) \Rightarrow (p \to r)$.

Example

Hypothetical Syllogism

Hypothetical Syllogism

Let p, q, r be propositions,

$$\begin{array}{c} p \rightarrow q \\ q \rightarrow r \\ \hline \therefore p \rightarrow r \end{array}$$

Observe that $((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$ is a tautology, hence we have $((p \rightarrow q) \wedge (q \rightarrow r)) \Rightarrow (p \rightarrow r)$.

Example

If Andre studies in Bandung, then he lives in Indonesia

If Andre lives in Indonesia, then he lives in planet Earth.

Hypothetical Syllogism

Hypothetical Syllogism

Let p, q, r be propositions,

$$\begin{array}{c} p \to q \\ q \to r \\ \hline \therefore p \to r \end{array}$$

Observe that $((p \to q) \wedge (q \to r)) \to (p \to r)$ is a tautology, hence we have $((p \to q) \wedge (q \to r)) \Rightarrow (p \to r)$.

Example

If Andre studies in Bandung, then he lives in Indonesia

If Andre lives in Indonesia, then he lives in planet Earth.

$\therefore$ If Andre studies in Bandung, then he lives in planet Earth.

Disjunctive Syllogism

Disjunctive Syllogism

Let p and q be propositions,

$$\begin{array}{c} p \vee q \\ \neg p \\ \hline \therefore q \end{array}$$

Disjunctive Syllogism

Disjunctive Syllogism

Let p and q be propositions,

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Observe that $((p \vee q) \wedge \neg p) \rightarrow q$ is a tautology, hence we have $((p \vee q) \wedge \neg p) \Rightarrow q$.

Example

Disjunctive Syllogism

Disjunctive Syllogism

Let p and q be propositions,

$$\begin{array}{l} p \vee q \\ \underline{\neg p \quad\quad} \\ \therefore q \end{array}$$

Observe that $((p \vee q) \wedge \neg p) \to q$ is a tautology, hence we have $((p \vee q) \wedge \neg p) \Rightarrow q$.

Example

Andre is a student or an employee.

Andre is not a student.

Disjunctive Syllogism

Disjunctive Syllogism

Let p and q be propositions,

$$\begin{array}{l} p \vee q \\ \neg p \\ \hline \therefore q \end{array}$$

Observe that $((p \vee q) \wedge \neg p) \rightarrow q$ is a tautology, hence we have $((p \vee q) \wedge \neg p) \Rightarrow q$.

Example

Andre is a student or an employee.

Andre is not a student.

∴ Andre is an employee.

Addition/ Disjunction Introduction

Addition/ Disjunction Introduction

Let p and q be propositions,

$$\frac{p}{\therefore p \vee q} \qquad\qquad \frac{q}{\therefore p \vee q}$$

Addition/ Disjunction Introduction

Addition/ Disjunction Introduction

Let p and q be propositions,

$$\frac{p}{\therefore p \vee q} \qquad\qquad \frac{q}{\therefore p \vee q}$$

Observe that $p \rightarrow (p \vee q)$ and $q \rightarrow (p \vee q)$ are tautology, hence we have $p \Rightarrow (p \vee q)$ and $q \Rightarrow (p \vee q)$.

Example

Addition/ Disjunction Introduction

Addition/ Disjunction Introduction

Let p and q be propositions,

$$\frac{p}{\therefore p \vee q} \qquad \frac{q}{\therefore p \vee q}$$

Observe that $p \rightarrow (p \vee q)$ and $q \rightarrow (p \vee q)$ are tautology, hence we have $p \Rightarrow (p \vee q)$ and $q \Rightarrow (p \vee q)$.

Example

Andre is a student.

Addition/ Disjunction Introduction

Addition/ Disjunction Introduction

Let p and q be propositions,

$$\frac{p}{\therefore p \vee q} \qquad \frac{q}{\therefore p \vee q}$$

Observe that $p \rightarrow (p \vee q)$ and $q \rightarrow (p \vee q)$ are tautology, hence we have $p \Rightarrow (p \vee q)$ and $q \Rightarrow (p \vee q)$.

Example

Andre is a student.

$\therefore$ Andre is a student or a janitor.

Simplification/ Conjunction Elimination

Simplification/ Conjunction Elimination

Let p and q be propositions,

$$\frac{p \wedge q}{\therefore p} \qquad\qquad \frac{p \wedge q}{\therefore q}$$

Simplification/ Conjunction Elimination

Simplification/ Conjunction Elimination

Let p and q be propositions,

$$\begin{array}{c} p \wedge q \\ \hline \therefore p \end{array} \qquad\qquad \begin{array}{c} p \wedge q \\ \hline \therefore q \end{array}$$

Observe that $(p \wedge q) \to p$ and $(p \wedge q) \to q$ are tautology, hence we have $(p \wedge q) \Rightarrow p$ and $(p \wedge q) \Rightarrow q$.

Example

Simplification/ Conjunction Elimination

Simplification/ Conjunction Elimination

Let p and q be propositions,

$$\frac{p \wedge q}{\therefore p} \qquad\qquad \frac{p \wedge q}{\therefore q}$$

Observe that $(p \wedge q) \rightarrow p$ and $(p \wedge q) \rightarrow q$ are tautology, hence we have $(p \wedge q) \Rightarrow p$ and $(p \wedge q) \Rightarrow q$.

Example

Andre studies at Tel-U and he lives in Bandung.

Simplification/ Conjunction Elimination

Simplification/ Conjunction Elimination

Let p and q be propositions,

$$\frac{p \wedge q}{\therefore p} \qquad \frac{p \wedge q}{\therefore q}$$

Observe that $(p \wedge q) \to p$ and $(p \wedge q) \to q$ are tautology, hence we have $(p \wedge q) \Rightarrow p$ and $(p \wedge q) \Rightarrow q$.

Example

Andre studies at Tel-U and he lives in Bandung.

$\therefore$ Andre studies at Tel-U.

Simplification/ Conjunction Elimination

Simplification/ Conjunction Elimination

Let p and q be propositions,

$$\frac{p \wedge q}{\therefore p} \qquad\qquad \frac{p \wedge q}{\therefore q}$$

Observe that $(p \wedge q) \rightarrow p$ and $(p \wedge q) \rightarrow q$ are tautology, hence we have $(p \wedge q) \Rightarrow p$ and $(p \wedge q) \Rightarrow q$.

Example

Andre studies at Tel-U and he lives in Bandung.

$\therefore$ Andre studies at Tel-U.

We can also infer the "Andre lives in Bandung".

Conjunction/ Conjunction Introduction

Conjunction/ Conjunction Introduction

Let p and q be propositions,

$$\begin{array}{l} p \\ q \\ \hline \therefore p \wedge q \end{array}$$

Conjunction/ Conjunction Introduction

Conjunction/ Conjunction Introduction

Let p and q be propositions,

$$\begin{array}{l} p \\ \underline{q \qquad\qquad} \\ \therefore p \wedge q \end{array}$$

Observe that $(p \wedge q) \to (p \wedge q)$ is a tautology, hence we have $(p \wedge q) \Rightarrow (p \wedge q)$.

Example

Conjunction/ Conjunction Introduction

Conjunction/ Conjunction Introduction

Let p and q be propositions,

$$\begin{array}{l} p \\ \underline{q \qquad\qquad} \\ \therefore p \wedge q \end{array}$$

Observe that $(p \wedge q) \rightarrow (p \wedge q)$ is a tautology, hence we have $(p \wedge q) \Rightarrow (p \wedge q)$.

Example

Andre studies at Tel-U.

Andre lives in Cimahi.

Conjunction/ Conjunction Introduction

Conjunction/ Conjunction Introduction

Let p and q be propositions,

$$\begin{array}{l} p \\ \underline{q \qquad\quad} \\ \therefore p \wedge q \end{array}$$

Observe that $(p \wedge q) \rightarrow (p \wedge q)$ is a tautology, hence we have $(p \wedge q) \Rightarrow (p \wedge q)$.

Example

Andre studies at Tel-U.

Andre lives in Cimahi.

$\therefore$ Andre studies at Tel-U and he lives in Cimahi.

Resolution

Resolution

Let p, q, r, be propositions,

$$\begin{array}{c} p \vee q \\ \neg p \vee r \\ \hline \therefore q \vee r \end{array}$$

Resolution

Resolution

Let p, q, r, be propositions,

$$\begin{array}{c} p \vee q \\ \neg p \vee r \\ \hline \therefore q \vee r \end{array}$$

Observe that $((p \vee q) \wedge (\neg p \vee r)) \rightarrow (q \vee r)$ is a tautology, hence we have $((p \vee q) \wedge (\neg p \vee r)) \Rightarrow (q \vee r)$.

Example

Resolution

Resolution

Let p, q, r, be propositions,

$$\begin{array}{c} p \vee q \\ \neg p \vee r \\ \hline \therefore q \vee r \end{array}$$

Observe that $((p \vee q) \wedge (\neg p \vee r)) \rightarrow (q \vee r)$ is a tautology, hence we have $((p \vee q) \wedge (\neg p \vee r)) \Rightarrow (q \vee r)$.

Example

Andre is a student or a janitor

Andre is not a student or he is a lecturer.

Resolution

Resolution

Let p, q, r, be propositions,

$$\begin{array}{c} p \vee q \\ \neg p \vee r \\ \hline \therefore q \vee r \end{array}$$

Observe that $((p \vee q) \wedge (\neg p \vee r)) \rightarrow (q \vee r)$ is a tautology, hence we have $((p \vee q) \wedge (\neg p \vee r)) \Rightarrow (q \vee r)$.

Example

Andre is a student or a janitor

Andre is not a student or he is a lecturer.

$\therefore$ Andre is a janitor or a lecturer.

- Resolution is an inference rule used by computer to perform *automatic reasoning*.
- In

$$\begin{array}{c} p \vee q \\ \neg p \vee r \\ \hline \therefore q \vee r \end{array}$$

  $q \vee r$ is called a *resolvent*.
- In resolution, all premises and conclusion are written in *clause* form.
- *Clause*: disjunction of propositional variables or negation of propositional variables (or their combination).

Contents

Propositional Inference: Exercise (1)

Exercise

Verify whether the premises $p \vee q \to r \wedge s$, $s \vee t \to u$, and p lead to the conclusion u.

Solution:

Propositional Inference: Exercise (1)

Exercise

Verify whether the premises $p \vee q \rightarrow r \wedge s$, $s \vee t \rightarrow u$, and p lead to the conclusion u.

Solution:

1. $p \vee q \rightarrow r \wedge s$ (premise)
2. $s \vee t \rightarrow u$ (premise)
3. p (premise)

Propositional Inference: Exercise (1)

Exercise

Verify whether the premises $p \vee q \rightarrow r \wedge s$, $s \vee t \rightarrow u$, and p lead to the conclusion u.

Solution:

1. $p \vee q \rightarrow r \wedge s$ (premise)
2. $s \vee t \rightarrow u$ (premise)
3. p (premise)
4. $p \vee q$ (addition from 3)

Propositional Inference: Exercise (1)

Exercise

Verify whether the premises $p \vee q \to r \wedge s$, $s \vee t \to u$, and p lead to the conclusion u.

Solution:

1. $p \vee q \to r \wedge s$ (premise)
2. $s \vee t \to u$ (premise)
3. p (premise)
4. $p \vee q$ (addition from 3)
5. $r \wedge s$ (modus ponens from 1 and 4)

Propositional Inference: Exercise (1)

Exercise

Verify whether the premises $p \vee q \rightarrow r \wedge s$, $s \vee t \rightarrow u$, and p lead to the conclusion u.

Solution:

1. $p \vee q \rightarrow r \wedge s$ (premise)
2. $s \vee t \rightarrow u$ (premise)
3. p (premise)
4. $p \vee q$ (addition from 3)
5. $r \wedge s$ (modus ponens from 1 and 4)
6. s (simplification from 5)

Propositional Inference: Exercise (1)

Exercise

Verify whether the premises $p \vee q \rightarrow r \wedge s$, $s \vee t \rightarrow u$, and p lead to the conclusion u.

Solution:

1. $p \vee q \rightarrow r \wedge s$ (premise)
2. $s \vee t \rightarrow u$ (premise)
3. p (premise)
4. $p \vee q$ (addition from 3)
5. $r \wedge s$ (modus ponens from 1 and 4)
6. s (simplification from 5)
7. $s \vee t$ (addition from 6)

Propositional Inference: Exercise (1)

Exercise

Verify whether the premises $p \vee q \rightarrow r \wedge s$, $s \vee t \rightarrow u$, and p lead to the conclusion u.

Solution:

1. $p \vee q \rightarrow r \wedge s$ (premise)
2. $s \vee t \rightarrow u$ (premise)
3. p (premise)
4. $p \vee q$ (addition from 3)
5. $r \wedge s$ (modus ponens from 1 and 4)
6. s (simplification from 5)
7. $s \vee t$ (addition from 6)
8. u (modus ponens from 2 and 7)

Propositional Inference: Exercise (2)

Exercise

Suppose we have the premises: "it is not sunny today and it is colder than yesterday", "if we will go swimming, then it must be sunny today", "if we do not go swimming, then we will go hiking", "if we go hiking, then we will be home by sunset".
Verify whether the premises lead to the conclusion: "we will be home by sunset".

Solution:

Propositional Inference: Exercise (2)

Exercise

Suppose we have the premises: "it is not sunny today and it is colder than yesterday", "if we will go swimming, then it must be sunny today", "if we do not go swimming, then we will go hiking", "if we go hiking, then we will be home by sunset".
Verify whether the premises lead to the conclusion: "we will be home by sunset".

Solution: suppose $p:$ "it is sunny today", $q:$ "it is colder than yesterday", $r:$ "we will go swimming", $s:$ "we will go hiking", $t:$ "we will be home by sunset".
Therefore, the premises can be rewritten as

Propositional Inference: Exercise (2)

Exercise

Suppose we have the premises: "it is not sunny today and it is colder than yesterday", "if we will go swimming, then it must be sunny today", "if we do not go swimming, then we will go hiking", "if we go hiking, then we will be home by sunset".
Verify whether the premises lead to the conclusion: "we will be home by sunset".

Solution: suppose p : "it is sunny today", q : "it is colder than yesterday", r : "we will go swimming", s : "we will go hiking", t : "we will be home by sunset".
Therefore, the premises can be rewritten as

$$\neg p \wedge q$$

Propositional Inference: Exercise (2)

Exercise

Suppose we have the premises: "it is not sunny today and it is colder than yesterday", "if we will go swimming, then it must be sunny today", "if we do not go swimming, then we will go hiking", "if we go hiking, then we will be home by sunset".
Verify whether the premises lead to the conclusion: "we will be home by sunset".

Solution: suppose p : "it is sunny today", q : "it is colder than yesterday", r : "we will go swimming", s : "we will go hiking", t : "we will be home by sunset".
Therefore, the premises can be rewritten as

$$\neg p \wedge q$$

$$r \rightarrow p$$

Propositional Inference: Exercise (2)

Exercise

Suppose we have the premises: "it is not sunny today and it is colder than yesterday", "if we will go swimming, then it must be sunny today", "if we do not go swimming, then we will go hiking", "if we go hiking, then we will be home by sunset".
Verify whether the premises lead to the conclusion: "we will be home by sunset".

Solution: suppose p : "it is sunny today", q : "it is colder than yesterday", r : "we will go swimming", s : "we will go hiking", t : "we will be home by sunset".
Therefore, the premises can be rewritten as

$$\neg p \wedge q$$

$$r \rightarrow p$$

$$\neg r \rightarrow s$$

Propositional Inference: Exercise (2)

Exercise

Suppose we have the premises: "it is not sunny today and it is colder than yesterday", "if we will go swimming, then it must be sunny today", "if we do not go swimming, then we will go hiking", "if we go hiking, then we will be home by sunset".
Verify whether the premises lead to the conclusion: "we will be home by sunset".

Solution: suppose p : "it is sunny today", q : "it is colder than yesterday", r : "we will go swimming", s : "we will go hiking", t : "we will be home by sunset".
Therefore, the premises can be rewritten as

$$\neg p \wedge q$$

$$r \rightarrow p$$

$$\neg r \rightarrow s$$

$$s \rightarrow t$$

We next verify whether these premises lead to the conclusion

Propositional Inference: Exercise (2)

Exercise

Suppose we have the premises: "it is not sunny today and it is colder than yesterday", "if we will go swimming, then it must be sunny today", "if we do not go swimming, then we will go hiking", "if we go hiking, then we will be home by sunset".
Verify whether the premises lead to the conclusion: "we will be home by sunset".

Solution: suppose p : "it is sunny today", q : "it is colder than yesterday", r : "we will go swimming", s : "we will go hiking", t : "we will be home by sunset".
Therefore, the premises can be rewritten as

$$\neg p \wedge q$$

$$r \rightarrow p$$

$$\neg r \rightarrow s$$

$$s \rightarrow t$$

We next verify whether these premises lead to the conclusion t by using valid propositional inference.

1. $\neg p \wedge q$ (premise)
2. $r \rightarrow p$ (premise)
3. $\neg r \rightarrow s$ (premise)
4. $s \rightarrow t$ (premise)

1. $\neg p \wedge q$ (premise)
2. $r \rightarrow p$ (premise)
3. $\neg r \rightarrow s$ (premise)
4. $s \rightarrow t$ (premise)
5. $\neg p$ (simplification from 1)

1. $\neg p \wedge q$ (premise)
2. $r \longrightarrow p$ (premise)
3. $\neg r \longrightarrow s$ (premise)
4. $s \longrightarrow t$ (premise)
5. $\neg p$ (simplification from 1)
6. $\neg r$ (modus tollens from 2 and 5)

1. $\neg p \wedge q$ (premise)
2. $r \longrightarrow p$ (premise)
3. $\neg r \longrightarrow s$ (premise)
4. $s \longrightarrow t$ (premise)
5. $\neg p$ (simplification from 1)
6. $\neg r$ (modus tollens from 2 and 5)
7. s (modus ponens from 3 and 6)

1. $\neg p \wedge q$ (premise)
2. $r \rightarrow p$ (premise)
3. $\neg r \rightarrow s$ (premise)
4. $s \rightarrow t$ (premise)
5. $\neg p$ (simplification from 1)
6. $\neg r$ (modus tollens from 2 and 5)
7. s (modus ponens from 3 and 6)
8. t (modus ponens from 4 and 7)

Therefore the conclusion t is *valid*.

Propositional Inference: Exercise (3)

Exercise

Suppose we have the statements: "If Alice sends an assignment email to Bob, then Bob will finish his homework", "if Alice doesn't send an assignment email to Bob, then he will play computer until midnight", "if Bob plays computer until midnight, then he will be sleepy in Mathematical Logic class".
Verify whether the statements lead to the conclusion: "if Bob doesn't finish his homework, then he will be sleepy in Mathematical Logic class".

Solution: suppose p : "Alice sends an assignment email to Bob", q : "Bob will finish his homework", r : "Bob will play computer until midnight ", and s : "Bob will be sleepy in Mathematical Logic class". The premises can be written as:

Solution: suppose p : "Alice sends an assignment email to Bob", q : "Bob will finish his homework", r : "Bob will play computer until midnight ", and s : "Bob will be sleepy in Mathematical Logic class". The premises can be written as:

$$p \rightarrow q$$

Solution: suppose p : "Alice sends an assignment email to Bob", q : "Bob will finish his homework", r : "Bob will play computer until midnight ", and s : "Bob will be sleepy in Mathematical Logic class". The premises can be written as:

$$\begin{aligned} p &\rightarrow q \\ \neg p &\rightarrow r \end{aligned}$$

Solution: suppose p : "Alice sends an assignment email to Bob", q : "Bob will finish his homework", r : "Bob will play computer until midnight ", and s : "Bob will be sleepy in Mathematical Logic class". The premises can be written as:

$$\begin{aligned} p &\rightarrow q \\ \neg p &\rightarrow r \\ r &\rightarrow s \end{aligned}$$

We next verify whether these premises lead to the conclusion

Solution: suppose p : "Alice sends an assignment email to Bob", q : "Bob will finish his homework", r : "Bob will play computer until midnight ", and s : "Bob will be sleepy in Mathematical Logic class". The premises can be written as:

$$\begin{aligned} p &\rightarrow q \\ \neg p &\rightarrow r \\ r &\rightarrow s \end{aligned}$$

We next verify whether these premises lead to the conclusion $\neg q \rightarrow s$ by using valid propositional inference.

Solution: suppose p : "Alice sends an assignment email to Bob", q : "Bob will finish his homework", r : "Bob will play computer until midnight ", and s : "Bob will be sleepy in Mathematical Logic class". The premises can be written as:

$$\begin{aligned} p &\rightarrow q \\ \neg p &\rightarrow r \\ r &\rightarrow s \end{aligned}$$

We next verify whether these premises lead to the conclusion $\neg q \rightarrow s$ by using valid propositional inference.

1. $p \rightarrow q$ (premise)
2. $\neg p \rightarrow r$ (premise)
3. $r \rightarrow s$ (premise)

Solution: suppose p : "Alice sends an assignment email to Bob", q : "Bob will finish his homework", r : "Bob will play computer until midnight ", and s : "Bob will be sleepy in Mathematical Logic class". The premises can be written as:

$$\begin{aligned} p &\rightarrow q \\ \neg p &\rightarrow r \\ r &\rightarrow s \end{aligned}$$

We next verify whether these premises lead to the conclusion $\neg q \rightarrow s$ by using valid propositional inference.

1. $p \rightarrow q$ (premise)
2. $\neg p \rightarrow r$ (premise)
3. $r \rightarrow s$ (premise)
4. $\neg q \rightarrow \neg p$ (contrapositive of 1)

Solution: suppose $p:$ "Alice sends an assignment email to Bob", $q:$ "Bob will finish his homework", $r:$ "Bob will play computer until midnight ", and $s:$ "Bob will be sleepy in Mathematical Logic class". The premises can be written as:

$$\begin{aligned} p &\rightarrow q \\ \neg p &\rightarrow r \\ r &\rightarrow s \end{aligned}$$

We next verify whether these premises lead to the conclusion $\neg q \rightarrow s$ by using valid propositional inference.

1. $p \rightarrow q$ (premise)
2. $\neg p \rightarrow r$ (premise)
3. $r \rightarrow s$ (premise)
4. $\neg q \rightarrow \neg p$ (contrapositive of 1)
5. $\neg q \rightarrow r$ (hypothetical syllogism from 4 and 2)

Solution: suppose p : "Alice sends an assignment email to Bob", q : "Bob will finish his homework", r : "Bob will play computer until midnight ", and s : "Bob will be sleepy in Mathematical Logic class". The premises can be written as:

$$\begin{aligned} p &\rightarrow q \\ \neg p &\rightarrow r \\ r &\rightarrow s \end{aligned}$$

We next verify whether these premises lead to the conclusion $\neg q \rightarrow s$ by using valid propositional inference.

1. $p \rightarrow q$ (premise)
2. $\neg p \rightarrow r$ (premise)
3. $r \rightarrow s$ (premise)
4. $\neg q \rightarrow \neg p$ (contrapositive of 1)
5. $\neg q \rightarrow r$ (hypothetical syllogism from 4 and 2)
6. $\neg q \rightarrow s$ (hypothetical syllogism from 5 and 3).

Therefore the conclusion $\neg q \rightarrow s$ is *valid*.

Propositional Inference: Exercise (4)

Exercise

Suppose we have the statements: "if today is raining and strong wind occurs, then there will be flood", "if there will be flood, then the people will suffer", "today strong wind occurs, but the people do not suffer".
Verify whether these statements lead to the conclusion: "today is not raining".

Solution:

Propositional Inference: Exercise (4)

Exercise

Suppose we have the statements: "if today is raining and strong wind occurs, then there will be flood", "if there will be flood, then the people will suffer", "today strong wind occurs, but the people do not suffer".
Verify whether these statements lead to the conclusion: "today is not raining".

Solution: suppose p : "today is raining", q : "today strong wind occurs", r : "there will be flood", s : "the people will suffer". The premises can be written as

Propositional Inference: Exercise (4)

Exercise

Suppose we have the statements: "if today is raining and strong wind occurs, then there will be flood", "if there will be flood, then the people will suffer", "today strong wind occurs, but the people do not suffer".
Verify whether these statements lead to the conclusion: "today is not raining".

Solution: suppose p : "today is raining", q : "today strong wind occurs", r : "there will be flood", s : "the people will suffer". The premises can be written as

$$p \wedge q \rightarrow r$$

Propositional Inference: Exercise (4)

Exercise

Suppose we have the statements: "if today is raining and strong wind occurs, then there will be flood", "if there will be flood, then the people will suffer", "today strong wind occurs, but the people do not suffer".
Verify whether these statements lead to the conclusion: "today is not raining".

Solution: suppose p : "today is raining", q : "today strong wind occurs", r : "there will be flood", s : "the people will suffer". The premises can be written as

$$p \wedge q \rightarrow r$$

$$r \rightarrow s$$

Propositional Inference: Exercise (4)

Exercise

Suppose we have the statements: "if today is raining and strong wind occurs, then there will be flood", "if there will be flood, then the people will suffer", "today strong wind occurs, but the people do not suffer".
Verify whether these statements lead to the conclusion: "today is not raining".

Solution: suppose p : "today is raining", q : "today strong wind occurs", r : "there will be flood", s : "the people will suffer". The premises can be written as

$$p \wedge q \rightarrow r$$

$$r \rightarrow s$$

$$q \wedge \neg s$$

We next verify whether the premises lead to a conclusion

Propositional Inference: Exercise (4)

Exercise

Suppose we have the statements: "if today is raining and strong wind occurs, then there will be flood", "if there will be flood, then the people will suffer", "today strong wind occurs, but the people do not suffer".
Verify whether these statements lead to the conclusion: "today is not raining".

Solution: suppose p : "today is raining", q : "today strong wind occurs", r : "there will be flood", s : "the people will suffer". The premises can be written as

$$p \wedge q \rightarrow r$$

$$r \rightarrow s$$

$$q \wedge \neg s$$

We next verify whether the premises lead to a conclusion $\neg p$ by using valid propositional inference.

1. $p \wedge q \rightarrow r$ (premise)
2. $r \rightarrow s$ (premise)
3. $q \wedge \neg s$ (premise)

1. $p \land q \to r$ (premise)
2. $r \to s$ (premise)
3. $q \land \neg s$ (premise)
4. $\neg s$ (simplification from 3)

1. $p \wedge q \rightarrow r$ (premise)
2. $r \rightarrow s$ (premise)
3. $q \wedge \neg s$ (premise)
4. $\neg s$ (simplification from 3)
5. $\neg r$ (modus tollens from 2 and 4)

1. $p \wedge q \rightarrow r$ (premise)
2. $r \rightarrow s$ (premise)
3. $q \wedge \neg s$ (premise)
4. $\neg s$ (simplification from 3)
5. $\neg r$ (modus tollens from 2 and 4)
6. $\neg (p \wedge q)$ (modus tollens from 1 and 5)

1. $p \wedge q \rightarrow r$ (premise)
2. $r \rightarrow s$ (premise)
3. $q \wedge \neg s$ (premise)
4. $\neg s$ (simplification from 3)
5. $\neg r$ (modus tollens from 2 and 4)
6. $\neg (p \wedge q)$ (modus tollens from 1 and 5)
7. $\neg p \vee \neg q$ (De Morgan's law from 6)

1. $p \wedge q \to r$ (premise)
2. $r \to s$ (premise)
3. $q \wedge \neg s$ (premise)
4. $\neg s$ (simplification from 3)
5. $\neg r$ (modus tollens from 2 and 4)
6. $\neg (p \wedge q)$ (modus tollens from 1 and 5)
7. $\neg p \vee \neg q$ (De Morgan's law from 6)
8. q (simplification from 3)

1. $p \wedge q \to r$ (premise)
2. $r \to s$ (premise)
3. $q \wedge \neg s$ (premise)
4. $\neg s$ (simplification from 3)
5. $\neg r$ (modus tollens from 2 and 4)
6. $\neg (p \wedge q)$ (modus tollens from 1 and 5)
7. $\neg p \vee \neg q$ (De Morgan's law from 6)
8. q (simplification from 3)
9. $\neg p$ (disjunctive syllogism from 7 and 8).

Therefore the conclusion $\neg p$ is *valid*.

Propositional Inference: Exercise (5)

Exercise

Verify whether the premises $(p \wedge q) \vee r$ and $r \rightarrow s$ lead to the conclusion $p \vee s$.

Solution:

Propositional Inference: Exercise (5)

Exercise

Verify whether the premises $(p \wedge q) \vee r$ and $r \rightarrow s$ lead to the conclusion $p \vee s$.

Solution:

1. $(p \wedge q) \vee r$ (premise)
2. $r \rightarrow s$ (premise)

Propositional Inference: Exercise (5)

Exercise

Verify whether the premises $(p \wedge q) \vee r$ and $r \rightarrow s$ lead to the conclusion $p \vee s$.

Solution:

1. $(p \wedge q) \vee r$ (premise)
2. $r \rightarrow s$ (premise)
3. $(p \vee r) \wedge (q \vee r)$ (distributive law from 1)

Propositional Inference: Exercise (5)

Exercise

Verify whether the premises $(p \wedge q) \vee r$ and $r \rightarrow s$ lead to the conclusion $p \vee s$.

Solution:

1. $(p \wedge q) \vee r$ (premise)
2. $r \rightarrow s$ (premise)
3. $(p \vee r) \wedge (q \vee r)$ (distributive law from 1)
4. $\neg r \vee s$ (equivalence $r \rightarrow s \equiv \neg r \vee s$ from 2)

Propositional Inference: Exercise (5)

Exercise

Verify whether the premises $(p \wedge q) \vee r$ and $r \rightarrow s$ lead to the conclusion $p \vee s$.

Solution:

1. $(p \wedge q) \vee r$ (premise)
2. $r \rightarrow s$ (premise)
3. $(p \vee r) \wedge (q \vee r)$ (distributive law from 1)
4. $\neg r \vee s$ (equivalence $r \rightarrow s \equiv \neg r \vee s$ from 2)
5. $p \vee r$ (simplification from 3)

Propositional Inference: Exercise (5)

Exercise

Verify whether the premises $(p \wedge q) \vee r$ and $r \rightarrow s$ lead to the conclusion $p \vee s$.

Solution:

1. $(p \wedge q) \vee r$ (premise)
2. $r \rightarrow s$ (premise)
3. $(p \vee r) \wedge (q \vee r)$ (distributive law from 1)
4. $\neg r \vee s$ (equivalence $r \rightarrow s \equiv \neg r \vee s$ from 2)
5. $p \vee r$ (simplification from 3)
6. $s \vee \neg r$ (commutative law from 4)

Propositional Inference: Exercise (5)

Exercise

Verify whether the premises $(p \wedge q) \vee r$ and $r \rightarrow s$ lead to the conclusion $p \vee s$.

Solution:

1. $(p \wedge q) \vee r$ (premise)
2. $r \rightarrow s$ (premise)
3. $(p \vee r) \wedge (q \vee r)$ (distributive law from 1)
4. $\neg r \vee s$ (equivalence $r \rightarrow s \equiv \neg r \vee s$ from 2)
5. $p \vee r$ (simplification from 3)
6. $s \vee \neg r$ (commutative law from 4)
7. $p \vee s$ (resolution from 5 and 6)

Propositional Inference: Exercise (6)

Exercise

Suppose we have the statements: "if today is snowing, then Alex will ski", "if today is not snowing, then Bryan will play hockey".
Verify whether these statements lead to the conclusion: "Alex will ski or Bryan will play hockey".

Solution:

Propositional Inference: Exercise (6)

Exercise

Suppose we have the statements: "if today is snowing, then Alex will ski", "if today is not snowing, then Bryan will play hockey".
Verify whether these statements lead to the conclusion: "Alex will ski or Bryan will play hockey".

Solution: suppose p : "today is snowing", q : "Alex will ski", r : "Bryan will play hockey". The premises can be written as

Propositional Inference: Exercise (6)

Exercise

Suppose we have the statements: "if today is snowing, then Alex will ski", "if today is not snowing, then Bryan will play hockey".
Verify whether these statements lead to the conclusion: "Alex will ski or Bryan will play hockey".

Solution: suppose p : "today is snowing", q : "Alex will ski", r : "Bryan will play hockey". The premises can be written as

$$p \quad \rightarrow \quad q$$

Propositional Inference: Exercise (6)

Exercise

Suppose we have the statements: "if today is snowing, then Alex will ski", "if today is not snowing, then Bryan will play hockey".
Verify whether these statements lead to the conclusion: "Alex will ski or Bryan will play hockey".

Solution: suppose p : "today is snowing", q : "Alex will ski", r : "Bryan will play hockey". The premises can be written as

$$\begin{array}{rcl} p & \rightarrow & q \\ \neg p & \rightarrow & r \end{array}$$

We next verify whether these premises lead to the conclusion

Propositional Inference: Exercise (6)

Exercise

Suppose we have the statements: "if today is snowing, then Alex will ski", "if today is not snowing, then Bryan will play hockey".
Verify whether these statements lead to the conclusion: "Alex will ski or Bryan will play hockey".

Solution: suppose p : "today is snowing", q : "Alex will ski", r : "Bryan will play hockey". The premises can be written as

$$\begin{aligned} p &\rightarrow q \\ \neg p &\rightarrow r \end{aligned}$$

We next verify whether these premises lead to the conclusion $q \vee r$ by using valid propositional inference.

1. $p \longrightarrow q$ (premise)
2. $\neg p \longrightarrow r$ (premise)

1. $p \longrightarrow q$ (premise)
2. $\neg p \longrightarrow r$ (premise)
3. $\neg p \vee q$ (equivalence $p \longrightarrow q \equiv \neg p \vee q$ from 1)

1. $p \rightarrow q$ (premise)
2. $\neg p \rightarrow r$ (premise)
3. $\neg p \vee q$ (equivalence $p \rightarrow q \equiv \neg p \vee q$ from 1)
4. $\neg\neg p \vee r$ (equivalence $\neg p \rightarrow r \equiv \neg\neg p \vee r$ from 2)

1. $p \rightarrow q$ (premise)
2. $\neg p \rightarrow r$ (premise)
3. $\neg p \vee q$ (equivalence $p \rightarrow q \equiv \neg p \vee q$ from 1)
4. $\neg\neg p \vee r$ (equivalence $\neg p \rightarrow r \equiv \neg\neg p \vee r$ from 2)
5. $p \vee r$ (double negation elimination $\neg\neg p$ from 4)

1. $p \rightarrow q$ (premise)
2. $\neg p \rightarrow r$ (premise)
3. $\neg p \vee q$ (equivalence $p \rightarrow q \equiv \neg p \vee q$ from 1)
4. $\neg\neg p \vee r$ (equivalence $\neg p \rightarrow r \equiv \neg\neg p \vee r$ from 2)
5. $p \vee r$ (double negation elimination $\neg\neg p$ from 4)
6. $q \vee r$ (resolution from 5 and 3).

Contents

Exercise: Verifying Argument Validity (1)

Exercise

Verify the validity of the following argument. Explain your answer.

If Andre studies regularly, then his final grade is A.
Andre's final grade is A.
Therefore, Andre studies regularly.

Solution:

Exercise: Verifying Argument Validity (1)

Exercise

Verify the validity of the following argument. Explain your answer.

If Andre studies regularly, then his final grade is A.
Andre's final grade is A.
Therefore, Andre studies regularly.

Solution:

- Suppose p : "Andre studies regularly" and q : "Andre's final grade is A".

Exercise: Verifying Argument Validity (1)

Exercise

Verify the validity of the following argument. Explain your answer.
If Andre studies regularly, then his final grade is A.
Andre's final grade is A.
Therefore, Andre studies regularly.

Solution:

- Suppose p : "Andre studies regularly" and q : "Andre's final grade is A".
- In the above reasoning, we have the premises $p \rightarrow q$ and q, and also the conclusion p.

Exercise: Verifying Argument Validity (1)

Exercise

Verify the validity of the following argument. Explain your answer.
If Andre studies regularly, then his final grade is A.
Andre's final grade is A.
Therefore, Andre studies regularly.

Solution:

- Suppose p : "Andre studies regularly" and q : "Andre's final grade is A".
- In the above reasoning, we have the premises $p \rightarrow q$ and q, and also the conclusion p.
- The above reasoning is not valid because $((p \rightarrow q) \wedge q) \rightarrow p$ is not a tautology (do you know why?).

Exercise: Verifying Argument Validity (1)

Exercise

Verify the validity of the following argument. Explain your answer.
If Andre studies regularly, then his final grade is A.
Andre's final grade is A.
Therefore, Andre studies regularly.

Solution:

- Suppose p : "Andre studies regularly" and q : "Andre's final grade is A".
- In the above reasoning, we have the premises $p \to q$ and q, and also the conclusion p.
- The above reasoning is not valid because $((p \to q) \wedge q) \to p$ is not a tautology (do you know why?).
- This type of incorrect reasoning is called the *fallacy of affirming the conclusion/ consequent* or *converse error*.

Exercise: Verifying Argument Validity (2)

Exercise

Verify the validity of the following argument. Explain your answer.

If Andre studies regularly, then his final grade is A.
Andre doesn't study regularly.
Therefore, Andre's final grade is not A.

Solution:

Exercise: Verifying Argument Validity (2)

Exercise

Verify the validity of the following argument. Explain your answer.

If Andre studies regularly, then his final grade is A.
Andre doesn't study regularly.
Therefore, Andre's final grade is not A.

Solution:

- Suppose p : "Andre studies regularly" and q : "Andre's final grade is A".

Exercise: Verifying Argument Validity (2)

Exercise

Verify the validity of the following argument. Explain your answer.
If Andre studies regularly, then his final grade is A.
Andre doesn't study regularly.
Therefore, Andre's final grade is not A.

Solution:

- Suppose p : "Andre studies regularly" and q : "Andre's final grade is A".
- In the above reasoning, we have the premises $p \rightarrow q$ and $\neg p$, and also the conclusion $\neg q$.

Exercise: Verifying Argument Validity (2)

Exercise

Verify the validity of the following argument. Explain your answer.
If Andre studies regularly, then his final grade is A.
Andre doesn't study regularly.
Therefore, Andre's final grade is not A.

Solution:

- Suppose p : "Andre studies regularly" and q : "Andre's final grade is A".
- In the above reasoning, we have the premises $p \rightarrow q$ and $\neg p$, and also the conclusion $\neg q$.
- The above reasoning is not valid because $((p \rightarrow q) \wedge \neg p) \rightarrow \neg q$ is not a tautology (do you know why?).

Exercise: Verifying Argument Validity (2)

Exercise

Verify the validity of the following argument. Explain your answer.
If Andre studies regularly, then his final grade is A.
Andre doesn't study regularly.
Therefore, Andre's final grade is not A.

Solution:

- Suppose p : "Andre studies regularly" and q : "Andre's final grade is A".
- In the above reasoning, we have the premises $p \to q$ and $\neg p$, and also the conclusion $\neg q$.
- The above reasoning is not valid because $((p \to q) \wedge \neg p) \to \neg q$ is not a tautology (do you know why?).
- This type of incorrect reasoning is called the *fallacy of denying the hypothesis/ antecedent* or the *inverse error*.

Exercise: Verifying Argument Validity (3)

Exercise

Verify the validity of the following argument.
If $\sqrt{2} > \frac{3}{2}$, then $\left(\sqrt{2}\right)^2 > \left(\frac{3}{2}\right)^2$. We know that $\sqrt{2} > \frac{3}{2}$. Consequently, $\left(\sqrt{2}\right)^2 > \left(\frac{3}{2}\right)^2$, or in other words $2 > \frac{9}{4}$.

Solution:

Exercise: Verifying Argument Validity (3)

Exercise

Verify the validity of the following argument.
If $\sqrt{2} > \frac{3}{2}$, then $\left(\sqrt{2}\right)^2 > \left(\frac{3}{2}\right)^2$. We know that $\sqrt{2} > \frac{3}{2}$. Consequently, $\left(\sqrt{2}\right)^2 > \left(\frac{3}{2}\right)^2$, or in other words $2 > \frac{9}{4}$.

Solution:

- Suppose $p : \sqrt{2} > \frac{3}{2}$ and $q : \left(\sqrt{2}\right)^2 > \left(\frac{3}{2}\right)^2$. Observe that q is also equivalent to $2 > \frac{9}{4}$.

Exercise: Verifying Argument Validity (3)

Exercise

Verify the validity of the following argument.
If $\sqrt{2} > \frac{3}{2}$, then $\left(\sqrt{2}\right)^2 > \left(\frac{3}{2}\right)^2$. We know that $\sqrt{2} > \frac{3}{2}$. Consequently, $\left(\sqrt{2}\right)^2 > \left(\frac{3}{2}\right)^2$, or in other words $2 > \frac{9}{4}$.

Solution:

- Suppose $p : \sqrt{2} > \frac{3}{2}$ and $q : \left(\sqrt{2}\right)^2 > \left(\frac{3}{2}\right)^2$. Observe that q is also equivalent to $2 > \frac{9}{4}$.
- The above reasoning has the premises $p \to q$ and p, and also the conclusion q.

Exercise: Verifying Argument Validity (3)

Exercise

Verify the validity of the following argument.
If $\sqrt{2} > \frac{3}{2}$, then $\left(\sqrt{2}\right)^2 > \left(\frac{3}{2}\right)^2$. We know that $\sqrt{2} > \frac{3}{2}$. Consequently, $\left(\sqrt{2}\right)^2 > \left(\frac{3}{2}\right)^2$, or in other words $2 > \frac{9}{4}$.

Solution:

- Suppose $p : \sqrt{2} > \frac{3}{2}$ and $q : \left(\sqrt{2}\right)^2 > \left(\frac{3}{2}\right)^2$. Observe that q is also equivalent to $2 > \frac{9}{4}$.
- The above reasoning has the premises $p \rightarrow q$ and p, and also the conclusion q.
- Therefore, the above reasoning is valid, because it is constructed from valid modus ponens rule.

Exercise: Verifying Argument Validity (3)

Exercise

Verify the validity of the following argument.
If $\sqrt{2} > \frac{3}{2}$, then $\left(\sqrt{2}\right)^2 > \left(\frac{3}{2}\right)^2$. We know that $\sqrt{2} > \frac{3}{2}$. Consequently, $\left(\sqrt{2}\right)^2 > \left(\frac{3}{2}\right)^2$, or in other words $2 > \frac{9}{4}$.

Solution:

- Suppose $p : \sqrt{2} > \frac{3}{2}$ and $q : \left(\sqrt{2}\right)^2 > \left(\frac{3}{2}\right)^2$. Observe that q is also equivalent to $2 > \frac{9}{4}$.
- The above reasoning has the premises $p \rightarrow q$ and p, and also the conclusion q.
- Therefore, the above reasoning is valid, because it is constructed from valid modus ponens rule.
- However, since p **false**, we cannot conclude that the reasoning conclusion is true.

Exercise: Verifying Argument Validity (3)

Exercise

Verify the validity of the following argument.
If $\sqrt{2} > \frac{3}{2}$, then $\left(\sqrt{2}\right)^2 > \left(\frac{3}{2}\right)^2$. We know that $\sqrt{2} > \frac{3}{2}$. Consequently, $\left(\sqrt{2}\right)^2 > \left(\frac{3}{2}\right)^2$, or in other words $2 > \frac{9}{4}$.

Solution:

- Suppose $p : \sqrt{2} > \frac{3}{2}$ and $q : \left(\sqrt{2}\right)^2 > \left(\frac{3}{2}\right)^2$. Observe that q is also equivalent to $2 > \frac{9}{4}$.
- The above reasoning has the premises $p \rightarrow q$ and p, and also the conclusion q.
- Therefore, the above reasoning is valid, because it is constructed from valid modus ponens rule.
- However, since p **false**, we cannot conclude that the reasoning conclusion is true.
- Moreover, we can also verify that the conclusion of the reasoning, that is $2 > \frac{9}{4}$, is **false**.