

# Propositional Logic 2: Interpretation and Semantics of Propositional Formulas – Laws of Logical Equivalences

Mathematical Logic – First Term 2023-2024

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September-October 2023

# Acknowledgements

This slide is compiled using the materials in the following sources:

- 1 *Discrete Mathematics and Its Applications* (Chapter 1), 8th Edition, 2019, by K. H. Rosen (primary reference).
- 2 *Discrete Mathematics with Applications* (Chapter 2), 5th Edition, 2018, by S. S. Epp.
- 3 *Logic in Computer Science: Modelling and Reasoning about Systems* (Chapter 1), 2nd Edition, 2004, by M. Huth and M. Ryan.
- 4 *Mathematical Logic for Computer Science* (Chapter 2, 3, 4), 2nd Edition, 2000, by M. Ben-Ari.
- 5 Discrete Mathematics 1 (2012) slides in Fasilkom UI by B. H. Widjaja.
- 6 Mathematical Logic slides in Telkom University by A. Rakhmatsyah and B. Purnama.

Some figures are excerpted from those sources. This slide is intended for internal academic purpose in SoC Telkom University. No slides are ever free from error nor incapable of being improved. Please convey your comments and corrections (if any) to [pleasedontspam@telkomuniversity.ac.id](mailto:<pleasedontspam>@telkomuniversity.ac.id).

# Contents

- 1 Interpretation and Semantics of Propositional Formulas
- 2 Propositional Formulas Based on Their Semantics
- 3 Formula Schema (Supplementary), Logical Consequence, and Logical Equivalence
- 4 Laws of Logical Equivalences

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# Interpretation

## Interpretation

An interpretation of a propositional formula is an assignment of truth value for that formula. The formula can be a compound proposition. For an atomic proposition, an interpretation is simply a mapping from a propositional variable to its truth. We denote the interpretation with following symbol:  $\mathcal{I}, \mathcal{I}_1, \mathcal{I}_2, \dots$

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- $\mathcal{I}_5(q) = \text{T}$  means  $q$  is interpreted to true by interpretation  $\mathcal{I}_5$

# Interpretation of Compound Propositions

- An interpretation of a proposition can be obtained **simply from the assignment of truth value of all atomic variables occur in that proposition.**
- The truth of a compound proposition can be determined from the truth of its constituent atomic propositions.
- For a complex formula, interpretation (also called semantics) is determined by observing the interpretation of all subformulas occur in that formula.

# Semantics Rules for Propositional Formulas

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Let  $A$  be a proposition and  $\mathcal{I}$  is an interpretation defined for each propositional atom occurs in  $A$ . The interpretation of  $A$  with respect to  $\mathcal{I}$  is defined recursively as follows:

- If  $A = p$  (an atomic proposition), then  $\mathcal{I}(A) = \mathcal{I}(p)$ , and its truth value is equal to the truth value of  $p$ .
- If  $A = \top$ , then  $\mathcal{I}(A) = \mathcal{I}(\top) =$

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- If  $A = \top$ , then  $\mathcal{I}(A) = \mathcal{I}(\top) = \top$ . Also, if  $A = \text{F}$ , then  $\mathcal{I}(A) = \mathcal{I}(\text{F}) =$

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- If  $A = p$  (an atomic proposition), then  $\mathcal{I}(A) = \mathcal{I}(p)$ , and its truth value is equal to the truth value of  $p$ .
- If  $A = \top$ , then  $\mathcal{I}(A) = \mathcal{I}(\top) = \top$ . Also, if  $A = \perp$ , then  $\mathcal{I}(A) = \mathcal{I}(\perp) = \perp$ .
- If  $A = \neg B$ , for some formula  $B$ , then

$$\mathcal{I}(A) =$$

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- If  $A = \text{T}$ , then  $\mathcal{I}(A) = \mathcal{I}(\text{T}) = \text{T}$ . Also, if  $A = \text{F}$ , then  $\mathcal{I}(A) = \mathcal{I}(\text{F}) = \text{F}$ .
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$$\mathcal{I}(A) = \mathcal{I}(\neg B) =$$

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- If  $A = \neg B$ , for some formula  $B$ , then
 
$$\mathcal{I}(A) = \mathcal{I}(\neg B) = \neg \mathcal{I}(B) = \begin{cases} \mathbf{T}, \end{cases}$$

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- If  $A = B \wedge C$ , for some formulas  $B$  and  $C$ , then
 
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- If  $A = \neg B$ , for some formula  $B$ , then
 
$$\mathcal{I}(A) = \mathcal{I}(\neg B) = \neg \mathcal{I}(B) = \begin{cases} \text{T}, & \text{if } \mathcal{I}(B) = \text{F} \\ \text{F}, & \text{if } \mathcal{I}(B) = \text{T} \end{cases} .$$
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- If  $A = B \oplus C$ , for some formulas  $B$  and  $C$ , then  
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- If  $A = B \rightarrow C$ , for some formulas  $B$  and  $C$ , then  $\mathcal{I}(A) = \mathcal{I}(B \rightarrow C) =$

$$\mathcal{I}(B) \rightarrow \mathcal{I}(C) = \begin{cases} \text{F,} & \text{if } \mathcal{I}(B) = \text{T but } \mathcal{I}(C) = \text{F} \\ \text{T,} & \text{otherwise} \end{cases}$$

- If  $A = B \oplus C$ , for some formulas  $B$  and  $C$ , then
 
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- If  $A = B \leftrightarrow C$ , for some formulas  $B$  and  $C$ , then
 
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## Example

Suppose  $A$  is the formula  $p \wedge q \rightarrow r$  and  $\mathcal{I}$  is an interpretation defined as:  $\mathcal{I}(p) = \text{F}$ ,  $\mathcal{I}(q) = \text{T}$ , and  $\mathcal{I}(r) = \text{F}$ . The interpretation of  $A$  under  $\mathcal{I}$ , i.e.,  $\mathcal{I}(A)$  is determined as follows:

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 &= (\mathcal{I}(p) \wedge \mathcal{I}(q)) \rightarrow \mathcal{I}(r) \\
 &= (\text{F} \wedge \text{T}) \rightarrow \text{F} =
 \end{aligned}$$

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 &= (\text{F} \wedge \text{T}) \rightarrow \text{F} = \text{F} \rightarrow \text{F} = \text{T}.
 \end{aligned}$$

This interpretation can also be determined as follows:

$$\mathcal{I}(p \wedge q \rightarrow r) =$$

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 &= (\mathcal{I}(p) \wedge \mathcal{I}(q)) \rightarrow \mathcal{I}(r) \\
 &= (\text{F} \wedge \text{T}) \rightarrow \text{F} = \text{F} \rightarrow \text{F} = \text{T}.
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This interpretation can also be determined as follows:

$$\mathcal{I}(p \wedge q \rightarrow r) = (\mathcal{I}(p) \wedge \mathcal{I}(q)) \rightarrow \mathcal{I}(r) = (\text{F} \wedge \text{T}) \rightarrow \text{F} =$$

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 \mathcal{I}(A) &= \mathcal{I}(p \wedge q \rightarrow r) \\
 &= \mathcal{I}((p \wedge q) \rightarrow r) \\
 &= \mathcal{I}(p \wedge q) \rightarrow \mathcal{I}(r) \\
 &= (\mathcal{I}(p) \wedge \mathcal{I}(q)) \rightarrow \mathcal{I}(r) \\
 &= (\text{F} \wedge \text{T}) \rightarrow \text{F} = \text{F} \rightarrow \text{F} = \text{T}.
 \end{aligned}$$

This interpretation can also be determined as follows:

$$\mathcal{I}(p \wedge q \rightarrow r) = (\mathcal{I}(p) \wedge \mathcal{I}(q)) \rightarrow \mathcal{I}(r) = (\text{F} \wedge \text{T}) \rightarrow \text{F} = \text{F} \rightarrow \text{F} =$$

## Example

Suppose  $A$  is the formula  $p \wedge q \rightarrow r$  and  $\mathcal{I}$  is an interpretation defined as:  $\mathcal{I}(p) = \text{F}$ ,  $\mathcal{I}(q) = \text{T}$ , and  $\mathcal{I}(r) = \text{F}$ . The interpretation of  $A$  under  $\mathcal{I}$ , i.e.,  $\mathcal{I}(A)$  is determined as follows:

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 \mathcal{I}(A) &= \mathcal{I}(p \wedge q \rightarrow r) \\
 &= \mathcal{I}((p \wedge q) \rightarrow r) \\
 &= \mathcal{I}(p \wedge q) \rightarrow \mathcal{I}(r) \\
 &= (\mathcal{I}(p) \wedge \mathcal{I}(q)) \rightarrow \mathcal{I}(r) \\
 &= (\text{F} \wedge \text{T}) \rightarrow \text{F} = \text{F} \rightarrow \text{F} = \text{T}.
 \end{aligned}$$

This interpretation can also be determined as follows:

$$\mathcal{I}(p \wedge q \rightarrow r) = (\mathcal{I}(p) \wedge \mathcal{I}(q)) \rightarrow \mathcal{I}(r) = (\text{F} \wedge \text{T}) \rightarrow \text{F} = \text{F} \rightarrow \text{F} = \text{T}.$$

## Exercise

Suppose  $\mathcal{I}$  and  $\mathcal{J}$  are two interpretations defined for atomic propositions as follows:  $\mathcal{I}(p) = \text{T}$ ,  $\mathcal{I}(q) = \text{F}$ ,  $\mathcal{I}(r) = \text{F}$ ,  $\mathcal{J}(p) = \text{F}$ ,  $\mathcal{J}(q) = \text{F}$ ,  $\mathcal{J}(r) = \text{T}$ . Determine the interpretation of  $p \rightarrow q \vee r$  and  $(p \rightarrow q) \vee (p \rightarrow r)$  under  $\mathcal{I}$  and  $\mathcal{J}$ .

Solution:

## Exercise

Suppose  $\mathcal{I}$  and  $\mathcal{J}$  are two interpretations defined for atomic propositions as follows:  $\mathcal{I}(p) = \text{T}$ ,  $\mathcal{I}(q) = \text{F}$ ,  $\mathcal{I}(r) = \text{F}$ ,  $\mathcal{J}(p) = \text{F}$ ,  $\mathcal{J}(q) = \text{F}$ ,  $\mathcal{J}(r) = \text{T}$ . Determine the interpretation of  $p \rightarrow q \vee r$  and  $(p \rightarrow q) \vee (p \rightarrow r)$  under  $\mathcal{I}$  and  $\mathcal{J}$ .

Solution:

$$\bullet \mathcal{I}(p \rightarrow q \vee r) =$$

## Exercise

Suppose  $\mathcal{I}$  and  $\mathcal{J}$  are two interpretations defined for atomic propositions as follows:  $\mathcal{I}(p) = \text{T}$ ,  $\mathcal{I}(q) = \text{F}$ ,  $\mathcal{I}(r) = \text{F}$ ,  $\mathcal{J}(p) = \text{F}$ ,  $\mathcal{J}(q) = \text{F}$ ,  $\mathcal{J}(r) = \text{T}$ . Determine the interpretation of  $p \rightarrow q \vee r$  and  $(p \rightarrow q) \vee (p \rightarrow r)$  under  $\mathcal{I}$  and  $\mathcal{J}$ .

Solution:

$$\bullet \mathcal{I}(p \rightarrow q \vee r) = \mathcal{I}(p \rightarrow (q \vee r)) = \mathcal{I}(p) \rightarrow (\mathcal{I}(q) \vee \mathcal{I}(r)) =$$

## Exercise

Suppose  $\mathcal{I}$  and  $\mathcal{J}$  are two interpretations defined for atomic propositions as follows:  $\mathcal{I}(p) = \text{T}$ ,  $\mathcal{I}(q) = \text{F}$ ,  $\mathcal{I}(r) = \text{F}$ ,  $\mathcal{J}(p) = \text{F}$ ,  $\mathcal{J}(q) = \text{F}$ ,  $\mathcal{J}(r) = \text{T}$ . Determine the interpretation of  $p \rightarrow q \vee r$  and  $(p \rightarrow q) \vee (p \rightarrow r)$  under  $\mathcal{I}$  and  $\mathcal{J}$ .

Solution:

$$\bullet \mathcal{I}(p \rightarrow q \vee r) = \mathcal{I}(p \rightarrow (q \vee r)) = \mathcal{I}(p) \rightarrow (\mathcal{I}(q) \vee \mathcal{I}(r)) = \text{T} \rightarrow (\text{F} \vee \text{F}) =$$

## Exercise

Suppose  $\mathcal{I}$  and  $\mathcal{J}$  are two interpretations defined for atomic propositions as follows:  $\mathcal{I}(p) = \text{T}$ ,  $\mathcal{I}(q) = \text{F}$ ,  $\mathcal{I}(r) = \text{F}$ ,  $\mathcal{J}(p) = \text{F}$ ,  $\mathcal{J}(q) = \text{F}$ ,  $\mathcal{J}(r) = \text{T}$ . Determine the interpretation of  $p \rightarrow q \vee r$  and  $(p \rightarrow q) \vee (p \rightarrow r)$  under  $\mathcal{I}$  and  $\mathcal{J}$ .

Solution:

$$\bullet \mathcal{I}(p \rightarrow q \vee r) = \mathcal{I}(p \rightarrow (q \vee r)) = \mathcal{I}(p) \rightarrow (\mathcal{I}(q) \vee \mathcal{I}(r)) = \text{T} \rightarrow (\text{F} \vee \text{F}) = \text{T} \rightarrow \text{F} =$$

## Exercise

Suppose  $\mathcal{I}$  and  $\mathcal{J}$  are two interpretations defined for atomic propositions as follows:  $\mathcal{I}(p) = \text{T}$ ,  $\mathcal{I}(q) = \text{F}$ ,  $\mathcal{I}(r) = \text{F}$ ,  $\mathcal{J}(p) = \text{F}$ ,  $\mathcal{J}(q) = \text{F}$ ,  $\mathcal{J}(r) = \text{T}$ . Determine the interpretation of  $p \rightarrow q \vee r$  and  $(p \rightarrow q) \vee (p \rightarrow r)$  under  $\mathcal{I}$  and  $\mathcal{J}$ .

Solution:

$$\bullet \mathcal{I}(p \rightarrow q \vee r) = \mathcal{I}(p \rightarrow (q \vee r)) = \mathcal{I}(p) \rightarrow (\mathcal{I}(q) \vee \mathcal{I}(r)) = \text{T} \rightarrow (\text{F} \vee \text{F}) = \text{T} \rightarrow \text{F} = \text{F}.$$

## Exercise

Suppose  $\mathcal{I}$  and  $\mathcal{J}$  are two interpretations defined for atomic propositions as follows:  $\mathcal{I}(p) = \text{T}$ ,  $\mathcal{I}(q) = \text{F}$ ,  $\mathcal{I}(r) = \text{F}$ ,  $\mathcal{J}(p) = \text{F}$ ,  $\mathcal{J}(q) = \text{F}$ ,  $\mathcal{J}(r) = \text{T}$ . Determine the interpretation of  $p \rightarrow q \vee r$  and  $(p \rightarrow q) \vee (p \rightarrow r)$  under  $\mathcal{I}$  and  $\mathcal{J}$ .

Solution:

$$\textcircled{1} \quad \mathcal{I}(p \rightarrow q \vee r) = \mathcal{I}(p \rightarrow (q \vee r)) = \mathcal{I}(p) \rightarrow (\mathcal{I}(q) \vee \mathcal{I}(r)) = \text{T} \rightarrow (\text{F} \vee \text{F}) = \text{T} \rightarrow \text{F} = \text{F}.$$

$$\textcircled{2} \quad \mathcal{J}(p \rightarrow q \vee r) =$$

## Exercise

Suppose  $\mathcal{I}$  and  $\mathcal{J}$  are two interpretations defined for atomic propositions as follows:  $\mathcal{I}(p) = \text{T}$ ,  $\mathcal{I}(q) = \text{F}$ ,  $\mathcal{I}(r) = \text{F}$ ,  $\mathcal{J}(p) = \text{F}$ ,  $\mathcal{J}(q) = \text{F}$ ,  $\mathcal{J}(r) = \text{T}$ . Determine the interpretation of  $p \rightarrow q \vee r$  and  $(p \rightarrow q) \vee (p \rightarrow r)$  under  $\mathcal{I}$  and  $\mathcal{J}$ .

Solution:

$$\textcircled{1} \quad \mathcal{I}(p \rightarrow q \vee r) = \mathcal{I}(p \rightarrow (q \vee r)) = \mathcal{I}(p) \rightarrow (\mathcal{I}(q) \vee \mathcal{I}(r)) = \text{T} \rightarrow (\text{F} \vee \text{F}) = \text{T} \rightarrow \text{F} = \text{F}.$$

$$\textcircled{2} \quad \mathcal{J}(p \rightarrow q \vee r) = \mathcal{J}(p \rightarrow (q \vee r)) = \mathcal{J}(p) \rightarrow (\mathcal{J}(q) \vee \mathcal{J}(r)) =$$

## Exercise

Suppose  $\mathcal{I}$  and  $\mathcal{J}$  are two interpretations defined for atomic propositions as follows:  $\mathcal{I}(p) = \text{T}$ ,  $\mathcal{I}(q) = \text{F}$ ,  $\mathcal{I}(r) = \text{F}$ ,  $\mathcal{J}(p) = \text{F}$ ,  $\mathcal{J}(q) = \text{F}$ ,  $\mathcal{J}(r) = \text{T}$ . Determine the interpretation of  $p \rightarrow q \vee r$  and  $(p \rightarrow q) \vee (p \rightarrow r)$  under  $\mathcal{I}$  and  $\mathcal{J}$ .

Solution:

- 1  $\mathcal{I}(p \rightarrow q \vee r) = \mathcal{I}(p \rightarrow (q \vee r)) = \mathcal{I}(p) \rightarrow (\mathcal{I}(q) \vee \mathcal{I}(r)) = \text{T} \rightarrow (\text{F} \vee \text{F}) = \text{T} \rightarrow \text{F} = \text{F}$ .
- 2  $\mathcal{J}(p \rightarrow q \vee r) = \mathcal{J}(p \rightarrow (q \vee r)) = \mathcal{J}(p) \rightarrow (\mathcal{J}(q) \vee \mathcal{J}(r)) = \text{F} \rightarrow (\text{F} \vee \text{T}) =$

## Exercise

Suppose  $\mathcal{I}$  and  $\mathcal{J}$  are two interpretations defined for atomic propositions as follows:  $\mathcal{I}(p) = \text{T}$ ,  $\mathcal{I}(q) = \text{F}$ ,  $\mathcal{I}(r) = \text{F}$ ,  $\mathcal{J}(p) = \text{F}$ ,  $\mathcal{J}(q) = \text{F}$ ,  $\mathcal{J}(r) = \text{T}$ . Determine the interpretation of  $p \rightarrow q \vee r$  and  $(p \rightarrow q) \vee (p \rightarrow r)$  under  $\mathcal{I}$  and  $\mathcal{J}$ .

Solution:

- 1  $\mathcal{I}(p \rightarrow q \vee r) = \mathcal{I}(p \rightarrow (q \vee r)) = \mathcal{I}(p) \rightarrow (\mathcal{I}(q) \vee \mathcal{I}(r)) = \text{T} \rightarrow (\text{F} \vee \text{F}) = \text{T} \rightarrow \text{F} = \text{F}$ .
- 2  $\mathcal{J}(p \rightarrow q \vee r) = \mathcal{J}(p \rightarrow (q \vee r)) = \mathcal{J}(p) \rightarrow (\mathcal{J}(q) \vee \mathcal{J}(r)) = \text{F} \rightarrow (\text{F} \vee \text{T}) = \text{F} \rightarrow \text{T} =$

## Exercise

Suppose  $\mathcal{I}$  and  $\mathcal{J}$  are two interpretations defined for atomic propositions as follows:  $\mathcal{I}(p) = \text{T}$ ,  $\mathcal{I}(q) = \text{F}$ ,  $\mathcal{I}(r) = \text{F}$ ,  $\mathcal{J}(p) = \text{F}$ ,  $\mathcal{J}(q) = \text{F}$ ,  $\mathcal{J}(r) = \text{T}$ . Determine the interpretation of  $p \rightarrow q \vee r$  and  $(p \rightarrow q) \vee (p \rightarrow r)$  under  $\mathcal{I}$  and  $\mathcal{J}$ .

Solution:

- 1  $\mathcal{I}(p \rightarrow q \vee r) = \mathcal{I}(p \rightarrow (q \vee r)) = \mathcal{I}(p) \rightarrow (\mathcal{I}(q) \vee \mathcal{I}(r)) = \text{T} \rightarrow (\text{F} \vee \text{F}) = \text{T} \rightarrow \text{F} = \text{F}$ .
- 2  $\mathcal{J}(p \rightarrow q \vee r) = \mathcal{J}(p \rightarrow (q \vee r)) = \mathcal{J}(p) \rightarrow (\mathcal{J}(q) \vee \mathcal{J}(r)) = \text{F} \rightarrow (\text{F} \vee \text{T}) = \text{F} \rightarrow \text{T} = \text{T}$ .

## Exercise

Suppose  $\mathcal{I}$  and  $\mathcal{J}$  are two interpretations defined for atomic propositions as follows:  $\mathcal{I}(p) = \mathbf{T}$ ,  $\mathcal{I}(q) = \mathbf{F}$ ,  $\mathcal{I}(r) = \mathbf{F}$ ,  $\mathcal{J}(p) = \mathbf{F}$ ,  $\mathcal{J}(q) = \mathbf{F}$ ,  $\mathcal{J}(r) = \mathbf{T}$ . Determine the interpretation of  $p \rightarrow q \vee r$  and  $(p \rightarrow q) \vee (p \rightarrow r)$  under  $\mathcal{I}$  and  $\mathcal{J}$ .

Solution:

- 1  $\mathcal{I}(p \rightarrow q \vee r) = \mathcal{I}(p \rightarrow (q \vee r)) = \mathcal{I}(p) \rightarrow (\mathcal{I}(q) \vee \mathcal{I}(r)) = \mathbf{T} \rightarrow (\mathbf{F} \vee \mathbf{F}) = \mathbf{T} \rightarrow \mathbf{F} = \mathbf{F}$ .
- 2  $\mathcal{J}(p \rightarrow q \vee r) = \mathcal{J}(p \rightarrow (q \vee r)) = \mathcal{J}(p) \rightarrow (\mathcal{J}(q) \vee \mathcal{J}(r)) = \mathbf{F} \rightarrow (\mathbf{F} \vee \mathbf{T}) = \mathbf{F} \rightarrow \mathbf{T} = \mathbf{T}$ .
- 3  $\mathcal{I}((p \rightarrow q) \vee (p \rightarrow r)) =$

## Exercise

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Solution:

- 1  $\mathcal{I}(p \rightarrow q \vee r) = \mathcal{I}(p \rightarrow (q \vee r)) = \mathcal{I}(p) \rightarrow (\mathcal{I}(q) \vee \mathcal{I}(r)) = \text{T} \rightarrow (\text{F} \vee \text{F}) = \text{T} \rightarrow \text{F} = \text{F}$ .
- 2  $\mathcal{J}(p \rightarrow q \vee r) = \mathcal{J}(p \rightarrow (q \vee r)) = \mathcal{J}(p) \rightarrow (\mathcal{J}(q) \vee \mathcal{J}(r)) = \text{F} \rightarrow (\text{F} \vee \text{T}) = \text{F} \rightarrow \text{T} = \text{T}$ .
- 3  $\mathcal{I}((p \rightarrow q) \vee (p \rightarrow r)) = \mathcal{I}(p \rightarrow q) \vee \mathcal{I}(p \rightarrow r) = (\mathcal{I}(p) \rightarrow \mathcal{I}(q)) \vee (\mathcal{I}(p) \rightarrow \mathcal{I}(r)) =$

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Solution:

- 1  $\mathcal{I}(p \rightarrow q \vee r) = \mathcal{I}(p \rightarrow (q \vee r)) = \mathcal{I}(p) \rightarrow (\mathcal{I}(q) \vee \mathcal{I}(r)) = \text{T} \rightarrow (\text{F} \vee \text{F}) = \text{T} \rightarrow \text{F} = \text{F}$ .
- 2  $\mathcal{J}(p \rightarrow q \vee r) = \mathcal{J}(p \rightarrow (q \vee r)) = \mathcal{J}(p) \rightarrow (\mathcal{J}(q) \vee \mathcal{J}(r)) = \text{F} \rightarrow (\text{F} \vee \text{T}) = \text{F} \rightarrow \text{T} = \text{T}$ .
- 3  $\mathcal{I}((p \rightarrow q) \vee (p \rightarrow r)) = \mathcal{I}(p \rightarrow q) \vee \mathcal{I}(p \rightarrow r) = (\mathcal{I}(p) \rightarrow \mathcal{I}(q)) \vee (\mathcal{I}(p) \rightarrow \mathcal{I}(r)) = (\text{T} \rightarrow \text{F}) \vee (\text{T} \rightarrow \text{F}) =$

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Suppose  $\mathcal{I}$  and  $\mathcal{J}$  are two interpretations defined for atomic propositions as follows:  $\mathcal{I}(p) = \text{T}$ ,  $\mathcal{I}(q) = \text{F}$ ,  $\mathcal{I}(r) = \text{F}$ ,  $\mathcal{J}(p) = \text{F}$ ,  $\mathcal{J}(q) = \text{F}$ ,  $\mathcal{J}(r) = \text{T}$ . Determine the interpretation of  $p \rightarrow q \vee r$  and  $(p \rightarrow q) \vee (p \rightarrow r)$  under  $\mathcal{I}$  and  $\mathcal{J}$ .

Solution:

- 1  $\mathcal{I}(p \rightarrow q \vee r) = \mathcal{I}(p \rightarrow (q \vee r)) = \mathcal{I}(p) \rightarrow (\mathcal{I}(q) \vee \mathcal{I}(r)) = \text{T} \rightarrow (\text{F} \vee \text{F}) = \text{T} \rightarrow \text{F} = \text{F}$ .
- 2  $\mathcal{J}(p \rightarrow q \vee r) = \mathcal{J}(p \rightarrow (q \vee r)) = \mathcal{J}(p) \rightarrow (\mathcal{J}(q) \vee \mathcal{J}(r)) = \text{F} \rightarrow (\text{F} \vee \text{T}) = \text{F} \rightarrow \text{T} = \text{T}$ .
- 3  $\mathcal{I}((p \rightarrow q) \vee (p \rightarrow r)) = \mathcal{I}(p \rightarrow q) \vee \mathcal{I}(p \rightarrow r) = (\mathcal{I}(p) \rightarrow \mathcal{I}(q)) \vee (\mathcal{I}(p) \rightarrow \mathcal{I}(r)) = (\text{T} \rightarrow \text{F}) \vee (\text{T} \rightarrow \text{F}) = \text{F} \vee \text{F} =$

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Suppose  $\mathcal{I}$  and  $\mathcal{J}$  are two interpretations defined for atomic propositions as follows:  $\mathcal{I}(p) = \text{T}$ ,  $\mathcal{I}(q) = \text{F}$ ,  $\mathcal{I}(r) = \text{F}$ ,  $\mathcal{J}(p) = \text{F}$ ,  $\mathcal{J}(q) = \text{F}$ ,  $\mathcal{J}(r) = \text{T}$ . Determine the interpretation of  $p \rightarrow q \vee r$  and  $(p \rightarrow q) \vee (p \rightarrow r)$  under  $\mathcal{I}$  and  $\mathcal{J}$ .

Solution:

- 1  $\mathcal{I}(p \rightarrow q \vee r) = \mathcal{I}(p \rightarrow (q \vee r)) = \mathcal{I}(p) \rightarrow (\mathcal{I}(q) \vee \mathcal{I}(r)) = \text{T} \rightarrow (\text{F} \vee \text{F}) = \text{T} \rightarrow \text{F} = \text{F}$ .
- 2  $\mathcal{J}(p \rightarrow q \vee r) = \mathcal{J}(p \rightarrow (q \vee r)) = \mathcal{J}(p) \rightarrow (\mathcal{J}(q) \vee \mathcal{J}(r)) = \text{F} \rightarrow (\text{F} \vee \text{T}) = \text{F} \rightarrow \text{T} = \text{T}$ .
- 3  $\mathcal{I}((p \rightarrow q) \vee (p \rightarrow r)) = \mathcal{I}(p \rightarrow q) \vee \mathcal{I}(p \rightarrow r) = (\mathcal{I}(p) \rightarrow \mathcal{I}(q)) \vee (\mathcal{I}(p) \rightarrow \mathcal{I}(r)) = (\text{T} \rightarrow \text{F}) \vee (\text{T} \rightarrow \text{F}) = \text{F} \vee \text{F} = \text{F}$ .

## Exercise

Suppose  $\mathcal{I}$  and  $\mathcal{J}$  are two interpretations defined for atomic propositions as follows:  $\mathcal{I}(p) = \text{T}$ ,  $\mathcal{I}(q) = \text{F}$ ,  $\mathcal{I}(r) = \text{F}$ ,  $\mathcal{J}(p) = \text{F}$ ,  $\mathcal{J}(q) = \text{F}$ ,  $\mathcal{J}(r) = \text{T}$ . Determine the interpretation of  $p \rightarrow q \vee r$  and  $(p \rightarrow q) \vee (p \rightarrow r)$  under  $\mathcal{I}$  and  $\mathcal{J}$ .

Solution:

- 1  $\mathcal{I}(p \rightarrow q \vee r) = \mathcal{I}(p \rightarrow (q \vee r)) = \mathcal{I}(p) \rightarrow (\mathcal{I}(q) \vee \mathcal{I}(r)) = \text{T} \rightarrow (\text{F} \vee \text{F}) = \text{T} \rightarrow \text{F} = \text{F}$ .
- 2  $\mathcal{J}(p \rightarrow q \vee r) = \mathcal{J}(p \rightarrow (q \vee r)) = \mathcal{J}(p) \rightarrow (\mathcal{J}(q) \vee \mathcal{J}(r)) = \text{F} \rightarrow (\text{F} \vee \text{T}) = \text{F} \rightarrow \text{T} = \text{T}$ .
- 3  $\mathcal{I}((p \rightarrow q) \vee (p \rightarrow r)) = \mathcal{I}(p \rightarrow q) \vee \mathcal{I}(p \rightarrow r) = (\mathcal{I}(p) \rightarrow \mathcal{I}(q)) \vee (\mathcal{I}(p) \rightarrow \mathcal{I}(r)) = (\text{T} \rightarrow \text{F}) \vee (\text{T} \rightarrow \text{F}) = \text{F} \vee \text{F} = \text{F}$ .
- 4  $\mathcal{J}((p \rightarrow q) \vee (p \rightarrow r)) =$

## Exercise

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Solution:

- 1  $\mathcal{I}(p \rightarrow q \vee r) = \mathcal{I}(p \rightarrow (q \vee r)) = \mathcal{I}(p) \rightarrow (\mathcal{I}(q) \vee \mathcal{I}(r)) = \text{T} \rightarrow (\text{F} \vee \text{F}) = \text{T} \rightarrow \text{F} = \text{F}$ .
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## Exercise

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Solution:

$$\textcircled{1} \quad \mathcal{I}(p \rightarrow q \vee r) = \mathcal{I}(p \rightarrow (q \vee r)) = \mathcal{I}(p) \rightarrow (\mathcal{I}(q) \vee \mathcal{I}(r)) = \text{T} \rightarrow (\text{F} \vee \text{F}) = \text{T} \rightarrow \text{F} = \text{F}.$$

$$\textcircled{2} \quad \mathcal{J}(p \rightarrow q \vee r) = \mathcal{J}(p \rightarrow (q \vee r)) = \mathcal{J}(p) \rightarrow (\mathcal{J}(q) \vee \mathcal{J}(r)) = \text{F} \rightarrow (\text{F} \vee \text{T}) = \text{F} \rightarrow \text{T} = \text{T}.$$

$$\textcircled{3} \quad \mathcal{I}((p \rightarrow q) \vee (p \rightarrow r)) = \mathcal{I}(p \rightarrow q) \vee \mathcal{I}(p \rightarrow r) = (\mathcal{I}(p) \rightarrow \mathcal{I}(q)) \vee (\mathcal{I}(p) \rightarrow \mathcal{I}(r)) = (\text{T} \rightarrow \text{F}) \vee (\text{T} \rightarrow \text{F}) = \text{F} \vee \text{F} = \text{F}.$$

$$\textcircled{4} \quad \mathcal{J}((p \rightarrow q) \vee (p \rightarrow r)) = \mathcal{J}(p \rightarrow q) \vee \mathcal{J}(p \rightarrow r) = (\mathcal{J}(p) \rightarrow \mathcal{J}(q)) \vee (\mathcal{J}(p) \rightarrow \mathcal{J}(r)) = (\text{F} \rightarrow \text{F}) \vee (\text{F} \rightarrow \text{T}) =$$

## Exercise

Suppose  $\mathcal{I}$  and  $\mathcal{J}$  are two interpretations defined for atomic propositions as follows:  $\mathcal{I}(p) = \text{T}$ ,  $\mathcal{I}(q) = \text{F}$ ,  $\mathcal{I}(r) = \text{F}$ ,  $\mathcal{J}(p) = \text{F}$ ,  $\mathcal{J}(q) = \text{F}$ ,  $\mathcal{J}(r) = \text{T}$ . Determine the interpretation of  $p \rightarrow q \vee r$  and  $(p \rightarrow q) \vee (p \rightarrow r)$  under  $\mathcal{I}$  and  $\mathcal{J}$ .

Solution:

$$\textcircled{1} \quad \mathcal{I}(p \rightarrow q \vee r) = \mathcal{I}(p \rightarrow (q \vee r)) = \mathcal{I}(p) \rightarrow (\mathcal{I}(q) \vee \mathcal{I}(r)) = \text{T} \rightarrow (\text{F} \vee \text{F}) = \text{T} \rightarrow \text{F} = \text{F}.$$

$$\textcircled{2} \quad \mathcal{J}(p \rightarrow q \vee r) = \mathcal{J}(p \rightarrow (q \vee r)) = \mathcal{J}(p) \rightarrow (\mathcal{J}(q) \vee \mathcal{J}(r)) = \text{F} \rightarrow (\text{F} \vee \text{T}) = \text{F} \rightarrow \text{T} = \text{T}.$$

$$\textcircled{3} \quad \mathcal{I}((p \rightarrow q) \vee (p \rightarrow r)) = \mathcal{I}(p \rightarrow q) \vee \mathcal{I}(p \rightarrow r) = (\mathcal{I}(p) \rightarrow \mathcal{I}(q)) \vee (\mathcal{I}(p) \rightarrow \mathcal{I}(r)) = (\text{T} \rightarrow \text{F}) \vee (\text{T} \rightarrow \text{F}) = \text{F} \vee \text{F} = \text{F}.$$

$$\textcircled{4} \quad \mathcal{J}((p \rightarrow q) \vee (p \rightarrow r)) = \mathcal{J}(p \rightarrow q) \vee \mathcal{J}(p \rightarrow r) = (\mathcal{J}(p) \rightarrow \mathcal{J}(q)) \vee (\mathcal{J}(p) \rightarrow \mathcal{J}(r)) = (\text{F} \rightarrow \text{F}) \vee (\text{F} \rightarrow \text{T}) = \text{T} \vee \text{T} =$$

## Exercise

Suppose  $\mathcal{I}$  and  $\mathcal{J}$  are two interpretations defined for atomic propositions as follows:  $\mathcal{I}(p) = \text{T}$ ,  $\mathcal{I}(q) = \text{F}$ ,  $\mathcal{I}(r) = \text{F}$ ,  $\mathcal{J}(p) = \text{F}$ ,  $\mathcal{J}(q) = \text{F}$ ,  $\mathcal{J}(r) = \text{T}$ . Determine the interpretation of  $p \rightarrow q \vee r$  and  $(p \rightarrow q) \vee (p \rightarrow r)$  under  $\mathcal{I}$  and  $\mathcal{J}$ .

Solution:

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# Interpretation and Truth Table

A single line in a truth table corresponds to an interpretation of a proposition.

Suppose  $A$  is the formula  $(p \vee \neg q) \rightarrow (p \wedge q)$ , the truth table of  $A$  can be obtained as follows:

Interpretation	$\mathcal{I}_k(\neg q)$	$\mathcal{I}_k(p \vee \neg q)$	$\mathcal{I}_k(p \wedge q)$	$\mathcal{I}_k(A)$
$\mathcal{I}_1(p) = \text{T}, \mathcal{I}_1(q) = \text{T}$				

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$\mathcal{I}_1(p) = \text{T}, \mathcal{I}_1(q) = \text{T}$	F			

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$\mathcal{I}_1(p) = \text{T}, \mathcal{I}_1(q) = \text{T}$	F	T	T	

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$\mathcal{I}_1(p) = \text{T}, \mathcal{I}_1(q) = \text{T}$	F	T	T	T
$\mathcal{I}_2(p) = \text{T}, \mathcal{I}_2(q) = \text{F}$				

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Interpretation	$\mathcal{I}_k(\neg q)$	$\mathcal{I}_k(p \vee \neg q)$	$\mathcal{I}_k(p \wedge q)$	$\mathcal{I}_k(A)$
$\mathcal{I}_1(p) = \text{T}, \mathcal{I}_1(q) = \text{T}$	F	T	T	T
$\mathcal{I}_2(p) = \text{T}, \mathcal{I}_2(q) = \text{F}$	T			

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$\mathcal{I}_1(p) = \text{T}, \mathcal{I}_1(q) = \text{T}$	F	T	T	T
$\mathcal{I}_2(p) = \text{T}, \mathcal{I}_2(q) = \text{F}$	T	T		

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$\mathcal{I}_1(p) = \text{T}, \mathcal{I}_1(q) = \text{T}$	F	T	T	T
$\mathcal{I}_2(p) = \text{T}, \mathcal{I}_2(q) = \text{F}$	T	T	F	F

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Interpretation	$\mathcal{I}_k(\neg q)$	$\mathcal{I}_k(p \vee \neg q)$	$\mathcal{I}_k(p \wedge q)$	$\mathcal{I}_k(A)$
$\mathcal{I}_1(p) = \text{T}, \mathcal{I}_1(q) = \text{T}$	F	T	T	T
$\mathcal{I}_2(p) = \text{T}, \mathcal{I}_2(q) = \text{F}$	T	T	F	F
$\mathcal{I}_3(p) = \text{F}, \mathcal{I}_3(q) = \text{T}$				

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$\mathcal{I}_1(p) = \text{T}, \mathcal{I}_1(q) = \text{T}$	F	T	T	T
$\mathcal{I}_2(p) = \text{T}, \mathcal{I}_2(q) = \text{F}$	T	T	F	F
$\mathcal{I}_3(p) = \text{F}, \mathcal{I}_3(q) = \text{T}$	F			

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$\mathcal{I}_1(p) = \text{T}, \mathcal{I}_1(q) = \text{T}$	F	T	T	T
$\mathcal{I}_2(p) = \text{T}, \mathcal{I}_2(q) = \text{F}$	T	T	F	F
$\mathcal{I}_3(p) = \text{F}, \mathcal{I}_3(q) = \text{T}$	F	F		

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$\mathcal{I}_1(p) = \text{T}, \mathcal{I}_1(q) = \text{T}$	F	T	T	T
$\mathcal{I}_2(p) = \text{T}, \mathcal{I}_2(q) = \text{F}$	T	T	F	F
$\mathcal{I}_3(p) = \text{F}, \mathcal{I}_3(q) = \text{T}$	F	F	F	F

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$\mathcal{I}_1(p) = \text{T}, \mathcal{I}_1(q) = \text{T}$	F	T	T	T
$\mathcal{I}_2(p) = \text{T}, \mathcal{I}_2(q) = \text{F}$	T	T	F	F
$\mathcal{I}_3(p) = \text{F}, \mathcal{I}_3(q) = \text{T}$	F	F	F	T
$\mathcal{I}_4(p) = \text{F}, \mathcal{I}_4(q) = \text{F}$				

# Interpretation and Truth Table

A single line in a truth table corresponds to an interpretation of a proposition.

Suppose  $A$  is the formula  $(p \vee \neg q) \rightarrow (p \wedge q)$ , the truth table of  $A$  can be obtained as follows:

Interpretation	$\mathcal{I}_k(\neg q)$	$\mathcal{I}_k(p \vee \neg q)$	$\mathcal{I}_k(p \wedge q)$	$\mathcal{I}_k(A)$
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$\mathcal{I}_2(p) = \text{T}, \mathcal{I}_2(q) = \text{F}$	T	T	F	F
$\mathcal{I}_3(p) = \text{F}, \mathcal{I}_3(q) = \text{T}$	F	F	F	T
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where  $k = 1, 2, 3, 4$ .

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How many lines are there in a truth table for verifying the truth of proposition with

- 1 3 different atomic propositions;
- 2 4 different atomic propositions;
- 3  $n$  different atomic propositions.

# Contents

- 1 Interpretation and Semantics of Propositional Formulas
- 2 Propositional Formulas Based on Their Semantics**
- 3 Formula Schema (Supplementary), Logical Consequence, and Logical Equivalence
- 4 Laws of Logical Equivalences

# Validity, Satisfiability, and Contradiction

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- 4  $A$  is a *contingency* iff  $A$  is neither valid nor contradictory.

## Consistent Set of Formulas

Suppose  $\{A_1, A_2, \dots, A_n\}$  is a collection of formulas. The set  $\{A_1, A_2, \dots, A_n\}$  is *consistent* or *mutually satisfiable* if there exists an interpretation  $\mathcal{I}$  such that

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The stipulation that  $\{A_1, A_2, \dots, A_n\}$  is consistent is equivalent to the condition that the conjunction

$$A_1 \wedge A_2 \wedge \dots \wedge A_n$$

is satisfiable.

# Example of Validity, Satisfiability, and Contradictory (1)

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Verify whether formula  $A$  defined as  $(p \oplus q) \vee (p \oplus \neg q)$  is valid, satisfiable, or contradictory.

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Since there exists an interpretation which makes  $A$  is true, then

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Since there exists an interpretation which makes  $A$  is true, then  $A$  is satisfiable.

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Since there exists an interpretation which makes  $A$  is true, then  $A$  is satisfiable.  
 Since every interpretation applied to  $A$  makes  $A$  is true, then

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Since every interpretation applied to  $A$  makes  $A$  is true, then  $A$  is valid.

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Since there exists an interpretation which makes  $A$  is true, then  $A$  is satisfiable.

Since every interpretation applied to  $A$  makes  $A$  is true, then  $A$  is valid.

Obviously,  $A$  is not a contradiction.

## Example of Validity, Satisfiability, and Contradictory (2)

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Verify whether formula  $A$  defined as  $(p \vee q) \wedge (\neg p \wedge \neg q)$  is valid, satisfiable, or contradictory.

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Solution: using truth table, consider that

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- 6 Since  $A$  is neither valid nor contradictory, then  $A$  is a contingency.

## Remark

In order to determine whether a formula  $A$  is satisfiable or not, **we do not always need to write down its truth table.**

## Problem

Is every valid formula also satisfiable?

# Proving Validity Without Truth Table (Supplementary)

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Given a formula  $A := \neg(p \wedge q \wedge r \wedge s) \rightarrow (\neg p \vee \neg q \vee \neg r \vee \neg s)$ , to determine whether  $A$  is valid, do we need to construct its truth table?

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- Truth table method is not always efficient.

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- Truth table method is not always efficient. The above formula  $A$  requires  $2^4 = 16$  rows in its truth table.
- We can prove the validity of a formula using *falsification* or *contradiction method*.
- In this method, **we first assume that the formula is not valid**, and then we try to derive that this assumption leads to contradictory statements (and therefore, our previous assumption is false).

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- 3 Recall that  $\mathcal{I}(A \rightarrow B) = \text{F}$  precisely when  $\mathcal{I}(A) = \text{T}$  and  $\mathcal{I}(B) = \text{F}$ , therefore we obtain  $\mathcal{I}(\neg p \vee \neg q) = \text{T}$  and  $\mathcal{I}(\neg(p \wedge q)) = \text{F}$ .
- 4 Since  $\mathcal{I}(\neg(p \wedge q)) = \text{F}$ , then  $\mathcal{I}(p \wedge q) = \text{T}$ , therefore it must be the case that  $\mathcal{I}(p) = \text{T}$  and  $\mathcal{I}(q) =$

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We want to prove that  $(\neg p \vee \neg q) \rightarrow \neg(p \wedge q)$  is valid by contradiction method.

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- 2 For convenience, we write  $A := \neg p \vee \neg q$  and  $B := \neg(p \wedge q)$ . Our formula can be rewritten as  $A \rightarrow B$ .
- 3 Recall that  $\mathcal{I}(A \rightarrow B) = \text{F}$  precisely when  $\mathcal{I}(A) = \text{T}$  and  $\mathcal{I}(B) = \text{F}$ , therefore we obtain  $\mathcal{I}(\neg p \vee \neg q) = \text{T}$  and  $\mathcal{I}(\neg(p \wedge q)) = \text{F}$ .
- 4 Since  $\mathcal{I}(\neg(p \wedge q)) = \text{F}$ , then  $\mathcal{I}(p \wedge q) = \text{T}$ , therefore it must be the case that  $\mathcal{I}(p) = \text{T}$  and  $\mathcal{I}(q) = \text{T}$ .
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- ⑤ From step (4), we get  $\mathcal{I}(\neg p) = \text{F}$  and  $\mathcal{I}(\neg q) = \text{F}$ , which makes  $\mathcal{I}(\neg p \vee \neg q) = \text{F}$ . This result contradicts to the previous  $\mathcal{I}(\neg p \vee \neg q) = \text{T}$  in step (3).
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- ② For convenience, we write  $A := \neg p \vee \neg q$  and  $B := \neg(p \wedge q)$ . Our formula can be rewritten as  $A \rightarrow B$ .
- ③ Recall that  $\mathcal{I}(A \rightarrow B) = \text{F}$  precisely when  $\mathcal{I}(A) = \text{T}$  and  $\mathcal{I}(B) = \text{F}$ , therefore we obtain  $\mathcal{I}(\neg p \vee \neg q) = \text{T}$  and  $\mathcal{I}(\neg(p \wedge q)) = \text{F}$ .
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- ⑤ From step (4), we get  $\mathcal{I}(\neg p) = \text{F}$  and  $\mathcal{I}(\neg q) = \text{F}$ , which makes  $\mathcal{I}(\neg p \vee \neg q) = \text{F}$ . This result contradicts to the previous  $\mathcal{I}(\neg p \vee \neg q) = \text{T}$  in step (3).
- ⑥ Accordingly, our assumption that  $(\neg p \vee \neg q) \rightarrow \neg(p \wedge q)$  is not valid was wrong.
- ⑦ Therefore,  $(\neg p \vee \neg q) \rightarrow \neg(p \wedge q)$  is a valid formula.

## Exercise

Show that  $(p \rightarrow q) \leftrightarrow (\neg p \vee q)$  is valid without using its truth table.

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For convenience, we write these conditions as  $\mathcal{I}_1(A) = \text{T}$  and  $\mathcal{I}_1(B) = \text{F}$ , and  $\mathcal{I}_2(A) = \text{F}$  and  $\mathcal{I}_2(B) = \text{T}$ .

For  $\mathcal{I}_1$ , we have  $\mathcal{I}_1(A) = \text{T}$  and  $\mathcal{I}_1(B) = \text{F}$ .

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- 2 This result contradicts the previous fact that  $\mathcal{I}_1(A) = \mathcal{I}_1(p \rightarrow q) = \text{T}$  for  $\mathcal{I}_1$ .

For  $\mathcal{I}_2$ , we have  $\mathcal{I}_2(A) = \text{F}$  and  $\mathcal{I}_2(B) = \text{T}$ .

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For  $\mathcal{I}_1$ , we have  $\mathcal{I}_1(A) = \text{T}$  and  $\mathcal{I}_1(B) = \text{F}$ .

- ① From  $\mathcal{I}_1(B) = \mathcal{I}_1(\neg p \vee q) = \text{F}$  we get  $\mathcal{I}_1(\neg p) = \mathcal{I}_1(q) = \text{F}$ , and thus  $\mathcal{I}_1(p) = \text{T}$ . From this result we obtain  $\mathcal{I}_1(p \rightarrow q) = \text{F}$ .
- ② This result contradicts the previous fact that  $\mathcal{I}_1(A) = \mathcal{I}_1(p \rightarrow q) = \text{T}$  for  $\mathcal{I}_1$ .

For  $\mathcal{I}_2$ , we have  $\mathcal{I}_2(A) = \text{F}$  and  $\mathcal{I}_2(B) = \text{T}$ .

- ① From  $\mathcal{I}_2(A) = \mathcal{I}_2(p \rightarrow q) = \text{F}$  we get  $\mathcal{I}_2(p) = \text{T}$  and  $\mathcal{I}_2(q) = \text{F}$ , and thus  $\mathcal{I}_2(\neg p) = \text{F}$ . From this result we obtain  $\mathcal{I}_2(\neg p \vee q) = \text{F}$ .
- ② This result contradicts the previous fact that  $\mathcal{I}_2(B) = \mathcal{I}_2(\neg p \vee q) = \text{T}$  for  $\mathcal{I}_2$ .

For  $\mathcal{I}_1$ , we have  $\mathcal{I}_1(A) = \text{T}$  and  $\mathcal{I}_1(B) = \text{F}$ .

- ① From  $\mathcal{I}_1(B) = \mathcal{I}_1(\neg p \vee q) = \text{F}$  we get  $\mathcal{I}_1(\neg p) = \mathcal{I}_1(q) = \text{F}$ , and thus  $\mathcal{I}_1(p) = \text{T}$ . From this result we obtain  $\mathcal{I}_1(p \rightarrow q) = \text{F}$ .
- ② This result contradicts the previous fact that  $\mathcal{I}_1(A) = \mathcal{I}_1(p \rightarrow q) = \text{T}$  for  $\mathcal{I}_1$ .

For  $\mathcal{I}_2$ , we have  $\mathcal{I}_2(A) = \text{F}$  and  $\mathcal{I}_2(B) = \text{T}$ .

- ① From  $\mathcal{I}_2(A) = \mathcal{I}_2(p \rightarrow q) = \text{F}$  we get  $\mathcal{I}_2(p) = \text{T}$  and  $\mathcal{I}_2(q) = \text{F}$ , and thus  $\mathcal{I}_2(\neg p) = \text{F}$ . From this result we obtain  $\mathcal{I}_2(\neg p \vee q) = \text{F}$ .
- ② This result contradicts the previous fact that  $\mathcal{I}_2(B) = \mathcal{I}_2(\neg p \vee q) = \text{T}$  for  $\mathcal{I}_2$ .

As a result, there is no interpretation  $\mathcal{I}$  that makes  $\mathcal{I}((p \rightarrow q) \leftrightarrow (\neg p \vee q)) = \text{F}$ .

For  $\mathcal{I}_1$ , we have  $\mathcal{I}_1(A) = \text{T}$  and  $\mathcal{I}_1(B) = \text{F}$ .

- ① From  $\mathcal{I}_1(B) = \mathcal{I}_1(\neg p \vee q) = \text{F}$  we get  $\mathcal{I}_1(\neg p) = \mathcal{I}_1(q) = \text{F}$ , and thus  $\mathcal{I}_1(p) = \text{T}$ . From this result we obtain  $\mathcal{I}_1(p \rightarrow q) = \text{F}$ .
- ② This result contradicts the previous fact that  $\mathcal{I}_1(A) = \mathcal{I}_1(p \rightarrow q) = \text{T}$  for  $\mathcal{I}_1$ .

For  $\mathcal{I}_2$ , we have  $\mathcal{I}_2(A) = \text{F}$  and  $\mathcal{I}_2(B) = \text{T}$ .

- ① From  $\mathcal{I}_2(A) = \mathcal{I}_2(p \rightarrow q) = \text{F}$  we get  $\mathcal{I}_2(p) = \text{T}$  and  $\mathcal{I}_2(q) = \text{F}$ , and thus  $\mathcal{I}_2(\neg p) = \text{F}$ . From this result we obtain  $\mathcal{I}_2(\neg p \vee q) = \text{F}$ .
- ② This result contradicts the previous fact that  $\mathcal{I}_2(B) = \mathcal{I}_2(\neg p \vee q) = \text{T}$  for  $\mathcal{I}_2$ .

As a result, there is no interpretation  $\mathcal{I}$  that makes  $\mathcal{I}((p \rightarrow q) \leftrightarrow (\neg p \vee q)) = \text{F}$ . Consequently, the formula  $(p \rightarrow q) \leftrightarrow (\neg p \vee q)$  is valid.

## Exercise

Show that  $\neg(p \wedge q \wedge r \wedge s) \rightarrow (\neg p \vee \neg q \vee \neg r \vee \neg s)$  is valid without using its truth table.

Solution:

## Exercise

Show that  $\neg(p \wedge q \wedge r \wedge s) \rightarrow (\neg p \vee \neg q \vee \neg r \vee \neg s)$  is valid without using its truth table.

Solution:

Suppose that  $\neg(p \wedge q \wedge r \wedge s) \rightarrow (\neg p \vee \neg q \vee \neg r \vee \neg s)$  is not valid,

## Exercise

Show that  $\neg(p \wedge q \wedge r \wedge s) \rightarrow (\neg p \vee \neg q \vee \neg r \vee \neg s)$  is valid without using its truth table.

Solution:

Suppose that  $\neg(p \wedge q \wedge r \wedge s) \rightarrow (\neg p \vee \neg q \vee \neg r \vee \neg s)$  is not valid, then there exists an interpretation  $\mathcal{I}$  such that

$$\mathcal{I}(\neg(p \wedge q \wedge r \wedge s) \rightarrow (\neg p \vee \neg q \vee \neg r \vee \neg s)) =$$

## Exercise

Show that  $\neg(p \wedge q \wedge r \wedge s) \rightarrow (\neg p \vee \neg q \vee \neg r \vee \neg s)$  is valid without using its truth table.

Solution:

Suppose that  $\neg(p \wedge q \wedge r \wedge s) \rightarrow (\neg p \vee \neg q \vee \neg r \vee \neg s)$  is not valid, then there exists an interpretation  $\mathcal{I}$  such that

$$\mathcal{I}(\neg(p \wedge q \wedge r \wedge s) \rightarrow (\neg p \vee \neg q \vee \neg r \vee \neg s)) = \mathbf{F}.$$

## Exercise

Show that  $\neg(p \wedge q \wedge r \wedge s) \rightarrow (\neg p \vee \neg q \vee \neg r \vee \neg s)$  is valid without using its truth table.

Solution:

Suppose that  $\neg(p \wedge q \wedge r \wedge s) \rightarrow (\neg p \vee \neg q \vee \neg r \vee \neg s)$  is not valid, then there exists an interpretation  $\mathcal{I}$  such that

$$\mathcal{I}(\neg(p \wedge q \wedge r \wedge s) \rightarrow (\neg p \vee \neg q \vee \neg r \vee \neg s)) = \text{F}.$$

For brevity, we write  $A := \neg(p \wedge q \wedge r \wedge s)$  and  $B := (\neg p \vee \neg q \vee \neg r \vee \neg s)$ .

## Exercise

Show that  $\neg(p \wedge q \wedge r \wedge s) \rightarrow (\neg p \vee \neg q \vee \neg r \vee \neg s)$  is valid without using its truth table.

Solution:

Suppose that  $\neg(p \wedge q \wedge r \wedge s) \rightarrow (\neg p \vee \neg q \vee \neg r \vee \neg s)$  is not valid, then there exists an interpretation  $\mathcal{I}$  such that

$$\mathcal{I}(\neg(p \wedge q \wedge r \wedge s) \rightarrow (\neg p \vee \neg q \vee \neg r \vee \neg s)) = \text{F}.$$

For brevity, we write  $A := \neg(p \wedge q \wedge r \wedge s)$  and  $B := (\neg p \vee \neg q \vee \neg r \vee \neg s)$ . Since  $\mathcal{I}(A \rightarrow B) = \text{F}$

## Exercise

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Solution:

Suppose that  $\neg(p \wedge q \wedge r \wedge s) \rightarrow (\neg p \vee \neg q \vee \neg r \vee \neg s)$  is not valid, then there exists an interpretation  $\mathcal{I}$  such that

$$\mathcal{I}(\neg(p \wedge q \wedge r \wedge s) \rightarrow (\neg p \vee \neg q \vee \neg r \vee \neg s)) = \text{F}.$$

For brevity, we write  $A := \neg(p \wedge q \wedge r \wedge s)$  and  $B := (\neg p \vee \neg q \vee \neg r \vee \neg s)$ . Since  $\mathcal{I}(A \rightarrow B) = \text{F}$  precisely when  $\mathcal{I}(A) =$

## Exercise

Show that  $\neg(p \wedge q \wedge r \wedge s) \rightarrow (\neg p \vee \neg q \vee \neg r \vee \neg s)$  is valid without using its truth table.

Solution:

Suppose that  $\neg(p \wedge q \wedge r \wedge s) \rightarrow (\neg p \vee \neg q \vee \neg r \vee \neg s)$  is not valid, then there exists an interpretation  $\mathcal{I}$  such that

$$\mathcal{I}(\neg(p \wedge q \wedge r \wedge s) \rightarrow (\neg p \vee \neg q \vee \neg r \vee \neg s)) = \text{F}.$$

For brevity, we write  $A := \neg(p \wedge q \wedge r \wedge s)$  and  $B := (\neg p \vee \neg q \vee \neg r \vee \neg s)$ . Since  $\mathcal{I}(A \rightarrow B) = \text{F}$  precisely when  $\mathcal{I}(A) = \text{T}$  and  $\mathcal{I}(B) =$

## Exercise

Show that  $\neg(p \wedge q \wedge r \wedge s) \rightarrow (\neg p \vee \neg q \vee \neg r \vee \neg s)$  is valid without using its truth table.

Solution:

Suppose that  $\neg(p \wedge q \wedge r \wedge s) \rightarrow (\neg p \vee \neg q \vee \neg r \vee \neg s)$  is not valid, then there exists an interpretation  $\mathcal{I}$  such that

$$\mathcal{I}(\neg(p \wedge q \wedge r \wedge s) \rightarrow (\neg p \vee \neg q \vee \neg r \vee \neg s)) = \text{F}.$$

For brevity, we write  $A := \neg(p \wedge q \wedge r \wedge s)$  and  $B := (\neg p \vee \neg q \vee \neg r \vee \neg s)$ . Since  $\mathcal{I}(A \rightarrow B) = \text{F}$  precisely when  $\mathcal{I}(A) = \text{T}$  and  $\mathcal{I}(B) = \text{F}$ , our assumption gives us:

$$\textcircled{1} \quad \mathcal{I}(B) = \mathcal{I}(\neg p \vee \neg q \vee \neg r \vee \neg s) =$$

## Exercise

Show that  $\neg(p \wedge q \wedge r \wedge s) \rightarrow (\neg p \vee \neg q \vee \neg r \vee \neg s)$  is valid without using its truth table.

Solution:

Suppose that  $\neg(p \wedge q \wedge r \wedge s) \rightarrow (\neg p \vee \neg q \vee \neg r \vee \neg s)$  is not valid, then there exists an interpretation  $\mathcal{I}$  such that

$$\mathcal{I}(\neg(p \wedge q \wedge r \wedge s) \rightarrow (\neg p \vee \neg q \vee \neg r \vee \neg s)) = \text{F}.$$

For brevity, we write  $A := \neg(p \wedge q \wedge r \wedge s)$  and  $B := (\neg p \vee \neg q \vee \neg r \vee \neg s)$ . Since  $\mathcal{I}(A \rightarrow B) = \text{F}$  precisely when  $\mathcal{I}(A) = \text{T}$  and  $\mathcal{I}(B) = \text{F}$ , our assumption gives us:

$$\begin{aligned} \textcircled{1} \quad \mathcal{I}(B) &= \mathcal{I}(\neg p \vee \neg q \vee \neg r \vee \neg s) = \text{F}, \text{ therefore} \\ \mathcal{I}(\neg p) &= \mathcal{I}(\neg q) = \mathcal{I}(\neg r) = \mathcal{I}(\neg s) = \end{aligned}$$

## Exercise

Show that  $\neg(p \wedge q \wedge r \wedge s) \rightarrow (\neg p \vee \neg q \vee \neg r \vee \neg s)$  is valid without using its truth table.

Solution:

Suppose that  $\neg(p \wedge q \wedge r \wedge s) \rightarrow (\neg p \vee \neg q \vee \neg r \vee \neg s)$  is not valid, then there exists an interpretation  $\mathcal{I}$  such that

$$\mathcal{I}(\neg(p \wedge q \wedge r \wedge s) \rightarrow (\neg p \vee \neg q \vee \neg r \vee \neg s)) = \text{F}.$$

For brevity, we write  $A := \neg(p \wedge q \wedge r \wedge s)$  and  $B := (\neg p \vee \neg q \vee \neg r \vee \neg s)$ . Since  $\mathcal{I}(A \rightarrow B) = \text{F}$  precisely when  $\mathcal{I}(A) = \text{T}$  and  $\mathcal{I}(B) = \text{F}$ , our assumption gives us:

$$\begin{aligned} \textcircled{1} \quad & \mathcal{I}(B) = \mathcal{I}(\neg p \vee \neg q \vee \neg r \vee \neg s) = \text{F}, \text{ therefore} \\ & \mathcal{I}(\neg p) = \mathcal{I}(\neg q) = \mathcal{I}(\neg r) = \mathcal{I}(\neg s) = \text{F}, \text{ and hence} \\ & \mathcal{I}(p) = \mathcal{I}(q) = \mathcal{I}(r) = \mathcal{I}(s) = \end{aligned}$$

## Exercise

Show that  $\neg(p \wedge q \wedge r \wedge s) \rightarrow (\neg p \vee \neg q \vee \neg r \vee \neg s)$  is valid without using its truth table.

Solution:

Suppose that  $\neg(p \wedge q \wedge r \wedge s) \rightarrow (\neg p \vee \neg q \vee \neg r \vee \neg s)$  is not valid, then there exists an interpretation  $\mathcal{I}$  such that

$$\mathcal{I}(\neg(p \wedge q \wedge r \wedge s) \rightarrow (\neg p \vee \neg q \vee \neg r \vee \neg s)) = \mathbf{F}.$$

For brevity, we write  $A := \neg(p \wedge q \wedge r \wedge s)$  and  $B := (\neg p \vee \neg q \vee \neg r \vee \neg s)$ . Since  $\mathcal{I}(A \rightarrow B) = \mathbf{F}$  precisely when  $\mathcal{I}(A) = \mathbf{T}$  and  $\mathcal{I}(B) = \mathbf{F}$ , our assumption gives us:

- 1  $\mathcal{I}(B) = \mathcal{I}(\neg p \vee \neg q \vee \neg r \vee \neg s) = \mathbf{F}$ , therefore  
 $\mathcal{I}(\neg p) = \mathcal{I}(\neg q) = \mathcal{I}(\neg r) = \mathcal{I}(\neg s) = \mathbf{F}$ , and hence  
 $\mathcal{I}(p) = \mathcal{I}(q) = \mathcal{I}(r) = \mathcal{I}(s) = \mathbf{T}$ .
- 2 From step (1), we obtain  $\mathcal{I}(p \wedge q \wedge r \wedge s) =$

## Exercise

Show that  $\neg(p \wedge q \wedge r \wedge s) \rightarrow (\neg p \vee \neg q \vee \neg r \vee \neg s)$  is valid without using its truth table.

Solution:

Suppose that  $\neg(p \wedge q \wedge r \wedge s) \rightarrow (\neg p \vee \neg q \vee \neg r \vee \neg s)$  is not valid, then there exists an interpretation  $\mathcal{I}$  such that

$$\mathcal{I}(\neg(p \wedge q \wedge r \wedge s) \rightarrow (\neg p \vee \neg q \vee \neg r \vee \neg s)) = \mathbf{F}.$$

For brevity, we write  $A := \neg(p \wedge q \wedge r \wedge s)$  and  $B := (\neg p \vee \neg q \vee \neg r \vee \neg s)$ . Since  $\mathcal{I}(A \rightarrow B) = \mathbf{F}$  precisely when  $\mathcal{I}(A) = \mathbf{T}$  and  $\mathcal{I}(B) = \mathbf{F}$ , our assumption gives us:

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 $\mathcal{I}(p) = \mathcal{I}(q) = \mathcal{I}(r) = \mathcal{I}(s) = \mathbf{T}$ .
- 2 From step (1), we obtain  $\mathcal{I}(p \wedge q \wedge r \wedge s) = \mathbf{T}$ , and thus  
 $\mathcal{I}(A) = \mathcal{I}(\neg(p \wedge q \wedge r \wedge s)) =$

## Exercise

Show that  $\neg(p \wedge q \wedge r \wedge s) \rightarrow (\neg p \vee \neg q \vee \neg r \vee \neg s)$  is valid without using its truth table.

Solution:

Suppose that  $\neg(p \wedge q \wedge r \wedge s) \rightarrow (\neg p \vee \neg q \vee \neg r \vee \neg s)$  is not valid, then there exists an interpretation  $\mathcal{I}$  such that

$$\mathcal{I}(\neg(p \wedge q \wedge r \wedge s) \rightarrow (\neg p \vee \neg q \vee \neg r \vee \neg s)) = \mathbf{F}.$$

For brevity, we write  $A := \neg(p \wedge q \wedge r \wedge s)$  and  $B := (\neg p \vee \neg q \vee \neg r \vee \neg s)$ . Since  $\mathcal{I}(A \rightarrow B) = \mathbf{F}$  precisely when  $\mathcal{I}(A) = \mathbf{T}$  and  $\mathcal{I}(B) = \mathbf{F}$ , our assumption gives us:

- ①  $\mathcal{I}(B) = \mathcal{I}(\neg p \vee \neg q \vee \neg r \vee \neg s) = \mathbf{F}$ , therefore  
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 $\mathcal{I}(p) = \mathcal{I}(q) = \mathcal{I}(r) = \mathcal{I}(s) = \mathbf{T}$ .
- ② From step (1), we obtain  $\mathcal{I}(p \wedge q \wedge r \wedge s) = \mathbf{T}$ , and thus  
 $\mathcal{I}(A) = \mathcal{I}(\neg(p \wedge q \wedge r \wedge s)) = \mathbf{F}$ .

## Exercise

Show that  $\neg(p \wedge q \wedge r \wedge s) \rightarrow (\neg p \vee \neg q \vee \neg r \vee \neg s)$  is valid without using its truth table.

Solution:

Suppose that  $\neg(p \wedge q \wedge r \wedge s) \rightarrow (\neg p \vee \neg q \vee \neg r \vee \neg s)$  is not valid, then there exists an interpretation  $\mathcal{I}$  such that

$$\mathcal{I}(\neg(p \wedge q \wedge r \wedge s) \rightarrow (\neg p \vee \neg q \vee \neg r \vee \neg s)) = \text{F}.$$

For brevity, we write  $A := \neg(p \wedge q \wedge r \wedge s)$  and  $B := (\neg p \vee \neg q \vee \neg r \vee \neg s)$ . Since  $\mathcal{I}(A \rightarrow B) = \text{F}$  precisely when  $\mathcal{I}(A) = \text{T}$  and  $\mathcal{I}(B) = \text{F}$ , our assumption gives us:

- ①  $\mathcal{I}(B) = \mathcal{I}(\neg p \vee \neg q \vee \neg r \vee \neg s) = \text{F}$ , therefore  
 $\mathcal{I}(\neg p) = \mathcal{I}(\neg q) = \mathcal{I}(\neg r) = \mathcal{I}(\neg s) = \text{F}$ , and hence  
 $\mathcal{I}(p) = \mathcal{I}(q) = \mathcal{I}(r) = \mathcal{I}(s) = \text{T}$ .
- ② From step (1), we obtain  $\mathcal{I}(p \wedge q \wedge r \wedge s) = \text{T}$ , and thus  
 $\mathcal{I}(A) = \mathcal{I}(\neg(p \wedge q \wedge r \wedge s)) = \text{F}$ .
- ③ The result in step (2) contradicts our previous supposition that  $\mathcal{I}(A) = \text{T}$ .

As a result, there is no interpretation  $\mathcal{I}$  satisfying  
 $\mathcal{I}(\neg(p \wedge q \wedge r \wedge s) \rightarrow (\neg p \vee \neg q \vee \neg r \vee \neg s)) = \mathbb{F}$ .

Therefore  $\neg(p \wedge q \wedge r \wedge s) \rightarrow (\neg p \vee \neg q \vee \neg r \vee \neg s)$  is valid.

## Exercise

Determine whether each of these formulas is valid, satisfiable, a contradiction, or a contingency.

$$1 \quad (p \rightarrow q \vee p \rightarrow r) \leftrightarrow (p \rightarrow q \wedge r)$$

$$2 \quad (p \wedge q \vee \neg r) \rightarrow (\neg(p \leftrightarrow r) \wedge \neg q)$$

$$3 \quad ((\neg p \wedge q) \wedge (r \rightarrow p) \wedge (r \rightarrow s) \wedge (s \rightarrow t)) \rightarrow t$$

$$4 \quad ((p \rightarrow q) \wedge (\neg p \rightarrow r) \wedge (r \rightarrow s)) \rightarrow (\neg q \rightarrow s)$$

$$5 \quad ((p \rightarrow q) \wedge (r \rightarrow s) \wedge (\neg p \rightarrow \neg s)) \rightarrow (r \rightarrow q)$$

$$6 \quad p \oplus q \rightarrow (p \wedge \neg q) \vee (\neg p \wedge q)$$

$$7 \quad p \oplus q \rightarrow (p \vee q) \wedge (\neg p \wedge \neg q)$$

$$8 \quad p \oplus q \rightarrow \neg(p \leftrightarrow q)$$

$$9 \quad \neg(p \leftrightarrow q) \leftrightarrow (p \wedge \neg q) \vee (\neg p \wedge q)$$

$$10 \quad (p \vee q) \wedge (\neg p \wedge \neg q) \leftrightarrow (p \wedge \neg q) \vee (\neg p \wedge q)$$

# Contents

- 1 Interpretation and Semantics of Propositional Formulas
- 2 Propositional Formulas Based on Their Semantics
- 3 Formula Schema (Supplementary), Logical Consequence, and Logical Equivalence**
- 4 Laws of Logical Equivalences

# Formula Schema (Supplementary)

Using truth tables, we can prove that each of these formulas is valid:

1  $A_1 := p \vee \neg p$

2  $A_2 := q \vee \neg q$

3  $A_3 := (p \rightarrow q) \vee \neg(p \rightarrow q)$

# Formula Schema (Supplementary)

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Observe that each of these formulas have a “similar” structure.

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In order to not prove each formula individually, we can investigate the formula schema  $A \vee \neg A$  instead.

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Observe that each of these formulas have a “similar” structure.

In order to not prove each formula individually, we can investigate the formula schema  $A \vee \neg A$  instead.

In  $A_1$ , we have  $p$  as formula  $A$ , in  $A_2$  we have  $q$  as formula  $A$ , and in  $A_3$  we have  $p \rightarrow q$  as formula  $A$ .

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Formula  $A_1$ ,  $A_2$ , and  $A_3$  which are obtained from replacing formula  $A$  with particular concrete formula are called as *instances* of formula schema  $A \vee \neg A$ .

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Observe that each of these formulas have a “similar” structure.

In order to not prove each formula individually, we can investigate the formula schema  $A \vee \neg A$  instead.

In  $A_1$ , we have  $p$  as formula  $A$ , in  $A_2$  we have  $q$  as formula  $A$ , and in  $A_3$  we have  $p \rightarrow q$  as formula  $A$ .

Formula  $A_1$ ,  $A_2$ , and  $A_3$  which are obtained from replacing formula  $A$  with particular concrete formula are called as *instances* of formula schema  $A \vee \neg A$ .

If formula schema  $A \vee \neg A$  is valid, then every instance of this formula schema is valid as well.

## Exercise

Show that following formulas are valid:

$$1 \quad \neg(p \wedge q) \rightarrow \neg p \vee \neg q$$

$$2 \quad \neg(p \wedge q \wedge r \wedge s) \rightarrow \neg(p \wedge q) \vee \neg(r \wedge s)$$

$$3 \quad \neg(p \wedge q \wedge r \wedge s \wedge t \wedge u) \rightarrow \neg(p \wedge q \wedge r) \vee \neg(s \wedge t \wedge u).$$

Solution:

## Exercise

Show that following formulas are valid:

$$① \quad \neg(p \wedge q) \rightarrow \neg p \vee \neg q$$

$$② \quad \neg(p \wedge q \wedge r \wedge s) \rightarrow \neg(p \wedge q) \vee \neg(r \wedge s)$$

$$③ \quad \neg(p \wedge q \wedge r \wedge s \wedge t \wedge u) \rightarrow \neg(p \wedge q \wedge r) \vee \neg(s \wedge t \wedge u).$$

Solution:

Observe that each of these formulas can be considered as formulas with identical schema, that is,

## Exercise

Show that following formulas are valid:

$$\textcircled{1} \quad \neg(p \wedge q) \rightarrow \neg p \vee \neg q$$

$$\textcircled{2} \quad \neg(p \wedge q \wedge r \wedge s) \rightarrow \neg(p \wedge q) \vee \neg(r \wedge s)$$

$$\textcircled{3} \quad \neg(p \wedge q \wedge r \wedge s \wedge t \wedge u) \rightarrow \neg(p \wedge q \wedge r) \vee \neg(s \wedge t \wedge u).$$

Solution:

Observe that each of these formulas can be considered as formulas with identical schema, that is,  $\neg(A \wedge B) \rightarrow \neg A \vee \neg B$ . For the first formula,  $A = p$  and  $B = q$ , for the second formula  $A =$

## Exercise

Show that following formulas are valid:

$$\textcircled{1} \quad \neg(p \wedge q) \rightarrow \neg p \vee \neg q$$

$$\textcircled{2} \quad \neg(p \wedge q \wedge r \wedge s) \rightarrow \neg(p \wedge q) \vee \neg(r \wedge s)$$

$$\textcircled{3} \quad \neg(p \wedge q \wedge r \wedge s \wedge t \wedge u) \rightarrow \neg(p \wedge q \wedge r) \vee \neg(s \wedge t \wedge u).$$

Solution:

Observe that each of these formulas can be considered as formulas with identical schema, that is,  $\neg(A \wedge B) \rightarrow \neg A \vee \neg B$ . For the first formula,  $A = p$  and  $B = q$ , for the second formula  $A = p \wedge q$  and  $B =$

## Exercise

Show that following formulas are valid:

$$① \quad \neg(p \wedge q) \rightarrow \neg p \vee \neg q$$

$$② \quad \neg(p \wedge q \wedge r \wedge s) \rightarrow \neg(p \wedge q) \vee \neg(r \wedge s)$$

$$③ \quad \neg(p \wedge q \wedge r \wedge s \wedge t \wedge u) \rightarrow \neg(p \wedge q \wedge r) \vee \neg(s \wedge t \wedge u).$$

Solution:

Observe that each of these formulas can be considered as formulas with identical schema, that is,  $\neg(A \wedge B) \rightarrow \neg A \vee \neg B$ . For the first formula,  $A = p$  and  $B = q$ , for the second formula  $A = p \wedge q$  and  $B = r \wedge s$ , and for the last formula  $A =$

## Exercise

Show that following formulas are valid:

$$① \quad \neg(p \wedge q) \rightarrow \neg p \vee \neg q$$

$$② \quad \neg(p \wedge q \wedge r \wedge s) \rightarrow \neg(p \wedge q) \vee \neg(r \wedge s)$$

$$③ \quad \neg(p \wedge q \wedge r \wedge s \wedge t \wedge u) \rightarrow \neg(p \wedge q \wedge r) \vee \neg(s \wedge t \wedge u).$$

Solution:

Observe that each of these formulas can be considered as formulas with identical schema, that is,  $\neg(A \wedge B) \rightarrow \neg A \vee \neg B$ . For the first formula,  $A = p$  and  $B = q$ , for the second formula  $A = p \wedge q$  and  $B = r \wedge s$ , and for the last formula  $A = p \wedge q \wedge r$  and  $B =$

## Exercise

Show that following formulas are valid:

$$① \quad \neg(p \wedge q) \rightarrow \neg p \vee \neg q$$

$$② \quad \neg(p \wedge q \wedge r \wedge s) \rightarrow \neg(p \wedge q) \vee \neg(r \wedge s)$$

$$③ \quad \neg(p \wedge q \wedge r \wedge s \wedge t \wedge u) \rightarrow \neg(p \wedge q \wedge r) \vee \neg(s \wedge t \wedge u).$$

Solution:

Observe that each of these formulas can be considered as formulas with identical schema, that is,  $\neg(A \wedge B) \rightarrow \neg A \vee \neg B$ . For the first formula,  $A = p$  and  $B = q$ , for the second formula  $A = p \wedge q$  and  $B = r \wedge s$ , and for the last formula  $A = p \wedge q \wedge r$  and  $B = s \wedge t \wedge u$ .

## Exercise

Show that following formulas are valid:

- 1  $\neg(p \wedge q) \rightarrow \neg p \vee \neg q$
- 2  $\neg(p \wedge q \wedge r \wedge s) \rightarrow \neg(p \wedge q) \vee \neg(r \wedge s)$
- 3  $\neg(p \wedge q \wedge r \wedge s \wedge t \wedge u) \rightarrow \neg(p \wedge q \wedge r) \vee \neg(s \wedge t \wedge u)$ .

Solution:

Observe that each of these formulas can be considered as formulas with identical schema, that is,  $\neg(A \wedge B) \rightarrow \neg A \vee \neg B$ . For the first formula,  $A = p$  and  $B = q$ , for the second formula  $A = p \wedge q$  and  $B = r \wedge s$ , and for the last formula  $A = p \wedge q \wedge r$  and  $B = s \wedge t \wedge u$ .

Accordingly, to prove that each of these formulas is valid, **it is sufficient to prove that formula schema  $\neg(A \wedge B) \rightarrow \neg A \vee \neg B$  is valid**. The prove can be carried out using contradiction method.

- 1 Suppose  $\neg(A \wedge B) \rightarrow \neg A \vee \neg B$  is not valid, then there is an interpretation  $\mathcal{I}$  such that  $\mathcal{I}(\neg(A \wedge B) \rightarrow \neg A \vee \neg B) =$

- 1 Suppose  $\neg(A \wedge B) \rightarrow \neg A \vee \neg B$  is not valid, then there is an interpretation  $\mathcal{I}$  such that  $\mathcal{I}(\neg(A \wedge B) \rightarrow \neg A \vee \neg B) = \text{F}$ .
- 2 As a consequence, we obtain  $\mathcal{I}(\neg(A \wedge B)) =$

- 1 Suppose  $\neg(A \wedge B) \rightarrow \neg A \vee \neg B$  is not valid, then there is an interpretation  $\mathcal{I}$  such that  $\mathcal{I}(\neg(A \wedge B) \rightarrow \neg A \vee \neg B) = \text{F}$ .
- 2 As a consequence, we obtain  $\mathcal{I}(\neg(A \wedge B)) = \text{T}$  and  $\mathcal{I}(\neg A \vee \neg B) =$

- 1 Suppose  $\neg(A \wedge B) \rightarrow \neg A \vee \neg B$  is not valid, then there is an interpretation  $\mathcal{I}$  such that  $\mathcal{I}(\neg(A \wedge B) \rightarrow \neg A \vee \neg B) = \text{F}$ .
- 2 As a consequence, we obtain  $\mathcal{I}(\neg(A \wedge B)) = \text{T}$  and  $\mathcal{I}(\neg A \vee \neg B) = \text{F}$ .
- 3 From  $\mathcal{I}(\neg A \vee \neg B) = \text{F}$  we obtain

- 1 Suppose  $\neg(A \wedge B) \rightarrow \neg A \vee \neg B$  is not valid, then there is an interpretation  $\mathcal{I}$  such that  $\mathcal{I}(\neg(A \wedge B) \rightarrow \neg A \vee \neg B) = \text{F}$ .
- 2 As a consequence, we obtain  $\mathcal{I}(\neg(A \wedge B)) = \text{T}$  and  $\mathcal{I}(\neg A \vee \neg B) = \text{F}$ .
- 3 From  $\mathcal{I}(\neg A \vee \neg B) = \text{F}$  we obtain  $\mathcal{I}(\neg A) = \mathcal{I}(\neg B) =$

- 1 Suppose  $\neg(A \wedge B) \rightarrow \neg A \vee \neg B$  is not valid, then there is an interpretation  $\mathcal{I}$  such that  $\mathcal{I}(\neg(A \wedge B) \rightarrow \neg A \vee \neg B) = \text{F}$ .
- 2 As a consequence, we obtain  $\mathcal{I}(\neg(A \wedge B)) = \text{T}$  and  $\mathcal{I}(\neg A \vee \neg B) = \text{F}$ .
- 3 From  $\mathcal{I}(\neg A \vee \neg B) = \text{F}$  we obtain  $\mathcal{I}(\neg A) = \mathcal{I}(\neg B) = \text{F}$ , and so  $\mathcal{I}(A) = \mathcal{I}(B) =$

- 1 Suppose  $\neg(A \wedge B) \rightarrow \neg A \vee \neg B$  is not valid, then there is an interpretation  $\mathcal{I}$  such that  $\mathcal{I}(\neg(A \wedge B) \rightarrow \neg A \vee \neg B) = \text{F}$ .
- 2 As a consequence, we obtain  $\mathcal{I}(\neg(A \wedge B)) = \text{T}$  and  $\mathcal{I}(\neg A \vee \neg B) = \text{F}$ .
- 3 From  $\mathcal{I}(\neg A \vee \neg B) = \text{F}$  we obtain  $\mathcal{I}(\neg A) = \mathcal{I}(\neg B) = \text{F}$ , and so  $\mathcal{I}(A) = \mathcal{I}(B) = \text{T}$ .
- 4 From step (3) we obtain  $\mathcal{I}(A \wedge B) =$

- 1 Suppose  $\neg(A \wedge B) \rightarrow \neg A \vee \neg B$  is not valid, then there is an interpretation  $\mathcal{I}$  such that  $\mathcal{I}(\neg(A \wedge B) \rightarrow \neg A \vee \neg B) = \text{F}$ .
- 2 As a consequence, we obtain  $\mathcal{I}(\neg(A \wedge B)) = \text{T}$  and  $\mathcal{I}(\neg A \vee \neg B) = \text{F}$ .
- 3 From  $\mathcal{I}(\neg A \vee \neg B) = \text{F}$  we obtain  $\mathcal{I}(\neg A) = \mathcal{I}(\neg B) = \text{F}$ , and so  $\mathcal{I}(A) = \mathcal{I}(B) = \text{T}$ .
- 4 From step (3) we obtain  $\mathcal{I}(A \wedge B) = \text{T}$ , hence  $\mathcal{I}(\neg(A \wedge B)) =$

- 1 Suppose  $\neg(A \wedge B) \rightarrow \neg A \vee \neg B$  is not valid, then there is an interpretation  $\mathcal{I}$  such that  $\mathcal{I}(\neg(A \wedge B) \rightarrow \neg A \vee \neg B) = \text{F}$ .
- 2 As a consequence, we obtain  $\mathcal{I}(\neg(A \wedge B)) = \text{T}$  and  $\mathcal{I}(\neg A \vee \neg B) = \text{F}$ .
- 3 From  $\mathcal{I}(\neg A \vee \neg B) = \text{F}$  we obtain  $\mathcal{I}(\neg A) = \mathcal{I}(\neg B) = \text{F}$ , and so  $\mathcal{I}(A) = \mathcal{I}(B) = \text{T}$ .
- 4 From step (3) we obtain  $\mathcal{I}(A \wedge B) = \text{T}$ , hence  $\mathcal{I}(\neg(A \wedge B)) = \text{F}$ .

- 1 Suppose  $\neg(A \wedge B) \rightarrow \neg A \vee \neg B$  is not valid, then there is an interpretation  $\mathcal{I}$  such that  $\mathcal{I}(\neg(A \wedge B) \rightarrow \neg A \vee \neg B) = \text{F}$ .
- 2 As a consequence, we obtain  $\mathcal{I}(\neg(A \wedge B)) = \text{T}$  and  $\mathcal{I}(\neg A \vee \neg B) = \text{F}$ .
- 3 From  $\mathcal{I}(\neg A \vee \neg B) = \text{F}$  we obtain  $\mathcal{I}(\neg A) = \mathcal{I}(\neg B) = \text{F}$ , and so  $\mathcal{I}(A) = \mathcal{I}(B) = \text{T}$ .
- 4 From step (3) we obtain  $\mathcal{I}(A \wedge B) = \text{T}$ , hence  $\mathcal{I}(\neg(A \wedge B)) = \text{F}$ . This result contradicts to  $\mathcal{I}(\neg(A \wedge B)) = \text{T}$  in step (2).

- 1 Suppose  $\neg(A \wedge B) \rightarrow \neg A \vee \neg B$  is not valid, then there is an interpretation  $\mathcal{I}$  such that  $\mathcal{I}(\neg(A \wedge B) \rightarrow \neg A \vee \neg B) = \text{F}$ .
- 2 As a consequence, we obtain  $\mathcal{I}(\neg(A \wedge B)) = \text{T}$  and  $\mathcal{I}(\neg A \vee \neg B) = \text{F}$ .
- 3 From  $\mathcal{I}(\neg A \vee \neg B) = \text{F}$  we obtain  $\mathcal{I}(\neg A) = \mathcal{I}(\neg B) = \text{F}$ , and so  $\mathcal{I}(A) = \mathcal{I}(B) = \text{T}$ .
- 4 From step (3) we obtain  $\mathcal{I}(A \wedge B) = \text{T}$ , hence  $\mathcal{I}(\neg(A \wedge B)) = \text{F}$ . This result contradicts to  $\mathcal{I}(\neg(A \wedge B)) = \text{T}$  in step (2).
- 5 Therefore, **there is no interpretation**  $\mathcal{I}$  such that  $\mathcal{I}(\neg(A \wedge B) \rightarrow \neg A \vee \neg B) = \text{F}$ .

- 1 Suppose  $\neg(A \wedge B) \rightarrow \neg A \vee \neg B$  is not valid, then there is an interpretation  $\mathcal{I}$  such that  $\mathcal{I}(\neg(A \wedge B) \rightarrow \neg A \vee \neg B) = F$ .
- 2 As a consequence, we obtain  $\mathcal{I}(\neg(A \wedge B)) = T$  and  $\mathcal{I}(\neg A \vee \neg B) = F$ .
- 3 From  $\mathcal{I}(\neg A \vee \neg B) = F$  we obtain  $\mathcal{I}(\neg A) = \mathcal{I}(\neg B) = F$ , and so  $\mathcal{I}(A) = \mathcal{I}(B) = T$ .
- 4 From step (3) we obtain  $\mathcal{I}(A \wedge B) = T$ , hence  $\mathcal{I}(\neg(A \wedge B)) = F$ . This result contradicts to  $\mathcal{I}(\neg(A \wedge B)) = T$  in step (2).
- 5 Therefore, **there is no interpretation**  $\mathcal{I}$  such that  $\mathcal{I}(\neg(A \wedge B) \rightarrow \neg A \vee \neg B) = F$ .
- 6 As a conclusion the formula schema  $\neg(A \wedge B) \rightarrow \neg A \vee \neg B$  is valid.

# Logical Consequence and Logical Equivalence

## Definition

Suppose  $A$  and  $B$  are two propositional formulas.  
Formula  $A$  and  $B$  are (*logically*) *equivalent* if the formula

$$A \leftrightarrow B$$

is a **tautology**. In this condition, we write  $A \equiv B$  or  $A \Leftrightarrow B$ .  
Formula  $B$  is said to be the (*logical*) *consequence* of  $A$  if the formula

$$A \rightarrow B$$

is a **tautology**. In this condition, we write  $A \Rightarrow B$ .

To verify a logical consequence or a logical equivalence of two formulas, we can use:

- truth table
- laws of logical equivalences

# Example of Logical Consequence (1)

## Exercise

Show that  $((p \rightarrow q) \wedge p) \Rightarrow q$ .

Solution:

# Example of Logical Consequence (1)

## Exercise

Show that  $((p \rightarrow q) \wedge p) \Rightarrow q$ .

Solution: we will show that  $((p \rightarrow q) \wedge p) \rightarrow q$  is a tautology by its truth table, observe that

$p$	$q$	$p \rightarrow q$	$(p \rightarrow q) \wedge p$	$((p \rightarrow q) \wedge p) \rightarrow q$
T	T			

# Example of Logical Consequence (1)

## Exercise

Show that  $((p \rightarrow q) \wedge p) \Rightarrow q$ .

Solution: we will show that  $((p \rightarrow q) \wedge p) \rightarrow q$  is a tautology by its truth table, observe that

$p$	$q$	$p \rightarrow q$	$(p \rightarrow q) \wedge p$	$((p \rightarrow q) \wedge p) \rightarrow q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

# Example of Logical Consequence (1)

## Exercise

Show that  $((p \rightarrow q) \wedge p) \Rightarrow q$ .

Solution: we will show that  $((p \rightarrow q) \wedge p) \rightarrow q$  is a tautology by its truth table, observe that

$p$	$q$	$p \rightarrow q$	$(p \rightarrow q) \wedge p$	$((p \rightarrow q) \wedge p) \rightarrow q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

The logical consequence  $((p \rightarrow q) \wedge p) \Rightarrow q$  is also known as **modus ponens** (we'll get back to this in the rule of inferences section).

## Example of Logical Consequence (2)

### Exercise

Show that  $((p \rightarrow q) \wedge \neg q) \Rightarrow \neg p$ .

Solution:

## Example of Logical Consequence (2)

### Exercise

Show that  $((p \rightarrow q) \wedge \neg q) \Rightarrow \neg p$ .

Solution: we will show that  $((p \rightarrow q) \wedge \neg q) \rightarrow \neg p$  is a tautology by its truth table, observe that

$p$	$q$	$\neg p$	$\neg q$	$p \rightarrow q$	$(p \rightarrow q) \wedge \neg q$	$\parallel$	$((p \rightarrow q) \wedge \neg q) \rightarrow \neg p$
T	T						

## Example of Logical Consequence (2)

### Exercise

Show that  $((p \rightarrow q) \wedge \neg q) \Rightarrow \neg p$ .

Solution: we will show that  $((p \rightarrow q) \wedge \neg q) \rightarrow \neg p$  is a tautology by its truth table, observe that

$p$	$q$	$\neg p$	$\neg q$	$p \rightarrow q$	$(p \rightarrow q) \wedge \neg q$	$((p \rightarrow q) \wedge \neg q) \rightarrow \neg p$
T	T	F	F	T	F	T
T	F	F	T	F	F	T
F	T	T	F	T	F	T
F	F	T	T	T	T	T

## Example of Logical Consequence (2)

### Exercise

Show that  $((p \rightarrow q) \wedge \neg q) \Rightarrow \neg p$ .

Solution: we will show that  $((p \rightarrow q) \wedge \neg q) \rightarrow \neg p$  is a tautology by its truth table, observe that

$p$	$q$	$\neg p$	$\neg q$	$p \rightarrow q$	$(p \rightarrow q) \wedge \neg q$	$((p \rightarrow q) \wedge \neg q) \rightarrow \neg p$
T	T	F	F	T	F	T
T	F	F	T	F	F	T
F	T	T	F	T	F	T
F	F	T	T	T	T	T

The logical consequence  $((p \rightarrow q) \wedge \neg q) \Rightarrow \neg p$  is also known as **modus tollens** (we'll get back to this in the rule of inferences section).

# Example of Logical Equivalence (1)

## Exercise

Show that  $p \rightarrow q \equiv \neg p \vee q$ .

Solution:

# Example of Logical Equivalence (1)

## Exercise

Show that  $p \rightarrow q \equiv \neg p \vee q$ .

Solution: we will show that  $(p \rightarrow q) \leftrightarrow (\neg p \vee q)$  is a tautology by its truth table, observe that

$p$	$q$	$\neg p$	$p \rightarrow q$	$\neg p \vee q$	$\parallel (p \rightarrow q) \leftrightarrow (\neg p \vee q)$
T	T				

# Example of Logical Equivalence (1)

## Exercise

Show that  $p \rightarrow q \equiv \neg p \vee q$ .

Solution: we will show that  $(p \rightarrow q) \leftrightarrow (\neg p \vee q)$  is a tautology by its truth table, observe that

$p$	$q$	$\neg p$	$p \rightarrow q$	$\neg p \vee q$	$(p \rightarrow q) \leftrightarrow (\neg p \vee q)$
T	T	F	T	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

# Example of Logical Equivalence (1)

## Exercise

Show that  $p \rightarrow q \equiv \neg p \vee q$ .

Solution: we will show that  $(p \rightarrow q) \leftrightarrow (\neg p \vee q)$  is a tautology by its truth table, observe that

$p$	$q$	$\neg p$	$p \rightarrow q$	$\neg p \vee q$	$(p \rightarrow q) \leftrightarrow (\neg p \vee q)$
T	T	F	T	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

Note that  $p \rightarrow q$  and  $\neg p \vee q$  have identical truth value in every row of the above truth table.

## Example of Logical Equivalence (2)

### Exercise

Show that  $\neg(p \wedge q) \equiv \neg p \vee \neg q$ .

Solution:

## Example of Logical Equivalence (2)

### Exercise

Show that  $\neg(p \wedge q) \equiv \neg p \vee \neg q$ .

Solution: we will show that  $\neg(p \wedge q) \leftrightarrow \neg p \vee \neg q$  is a tautology by its truth table, observe that

$p$	$q$	$\neg p$	$\neg q$	$p \wedge q$	$\neg(p \wedge q)$	$\neg p \vee \neg q$	$\neg(p \wedge q) \leftrightarrow \neg p \vee \neg q$
T	T						

## Example of Logical Equivalence (2)

### Exercise

Show that  $\neg(p \wedge q) \equiv \neg p \vee \neg q$ .

Solution: we will show that  $\neg(p \wedge q) \leftrightarrow \neg p \vee \neg q$  is a tautology by its truth table, observe that

$p$	$q$	$\neg p$	$\neg q$	$p \wedge q$	$\neg(p \wedge q)$	$\neg p \vee \neg q$	$\neg(p \wedge q) \leftrightarrow \neg p \vee \neg q$
T	T	F	F	T	F	F	T
T	F	F	T	F	T	T	T
F	T	T	F	F	T	T	T
F	F	T	T	F	T	T	T

## Example of Logical Equivalence (2)

### Exercise

Show that  $\neg(p \wedge q) \equiv \neg p \vee \neg q$ .

Solution: we will show that  $\neg(p \wedge q) \leftrightarrow \neg p \vee \neg q$  is a tautology by its truth table, observe that

$p$	$q$	$\neg p$	$\neg q$	$p \wedge q$	$\neg(p \wedge q)$	$\neg p \vee \neg q$	$\neg(p \wedge q) \leftrightarrow \neg p \vee \neg q$
T	T	F	F	T	F	F	T
T	F	F	T	F	T	T	T
F	T	T	F	F	T	T	T
F	F	T	T	F	T	T	T

Note that  $\neg(p \wedge q)$  and  $\neg p \vee \neg q$  have identical truth value in every row of the above truth table. The logical equivalence  $\neg(p \wedge q) \equiv \neg p \vee \neg q$  is also known as **De Morgan's law**.

# Contents

- 1 Interpretation and Semantics of Propositional Formulas
- 2 Propositional Formulas Based on Their Semantics
- 3 Formula Schema (Supplementary), Logical Consequence, and Logical Equivalence
- 4 Laws of Logical Equivalences**

# Logical Equivalences Concerning $\neg$ , $\wedge$ , and $\vee$

$p \wedge \mathbf{T} \equiv p$ $p \vee \mathbf{F} \equiv p$	Identity laws
$p \vee \mathbf{T} \equiv \mathbf{T}$ $p \wedge \mathbf{F} \equiv \mathbf{F}$	Domination laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent laws
$\neg(\neg p) \equiv p$	Double negation law
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative laws

$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$ $(p \vee q) \vee r \equiv p \vee (q \vee r)$	Associative laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive laws
$\neg (p \wedge q) \equiv \neg p \vee \neg q$ $\neg (p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption laws
$p \vee \neg p \equiv \mathbf{T}$ $p \wedge \neg p \equiv \mathbf{F}$	Negation laws

By associative laws, the parentheses in formulas  $(p \wedge q) \wedge r$ ,  $p \wedge (q \wedge r)$ ,  $(p \vee q) \vee r$ , and  $p \vee (q \vee r)$  can be omitted and these formula can be rewritten identically as  $p \wedge q \wedge r$  (for conjunctive form) and  $p \vee q \vee r$  (for disjunctive form).

# Logical Equivalences Concerning $\rightarrow$ and $\leftrightarrow$

$$p \rightarrow q \equiv \neg p \vee q$$

$$p \rightarrow q \equiv \neg q \rightarrow \neg p$$

$$\neg(p \rightarrow q) \equiv p \wedge \neg q$$

$$(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r)$$

$$(p \rightarrow r) \wedge (q \rightarrow r) \equiv (p \wedge q) \rightarrow r$$

$$(p \rightarrow r) \vee (q \rightarrow r) \equiv (p \vee q) \rightarrow r$$

$$(p \rightarrow q) \vee (p \rightarrow r) \equiv p \rightarrow (q \vee r)$$

$$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$$

$$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$$

$$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$$

$$\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$$

# Proving Equivalence Using Laws of Equivalences (1)

## Exercise

Show that  $p \wedge q \rightarrow r \equiv p \rightarrow (q \rightarrow r)$  using the laws of logical equivalences.

Solution:

# Proving Equivalence Using Laws of Equivalences (1)

## Exercise

Show that  $p \wedge q \rightarrow r \equiv p \rightarrow (q \rightarrow r)$  using the laws of logical equivalences.

Solution:

$$\begin{aligned} p \wedge q \rightarrow r &\equiv (p \wedge q) \rightarrow r && \text{(precedences of } \wedge \text{ and } \rightarrow) \\ &\equiv \end{aligned}$$

# Proving Equivalence Using Laws of Equivalences (1)

## Exercise

Show that  $p \wedge q \rightarrow r \equiv p \rightarrow (q \rightarrow r)$  using the laws of logical equivalences.

Solution:

$$\begin{aligned}
 p \wedge q \rightarrow r &\equiv (p \wedge q) \rightarrow r && \text{(precedences of } \wedge \text{ and } \rightarrow) \\
 &\equiv \neg(p \wedge q) \vee r && \text{(equivalence } A \rightarrow B \equiv \neg A \vee B) \\
 &\equiv
 \end{aligned}$$

# Proving Equivalence Using Laws of Equivalences (1)

## Exercise

Show that  $p \wedge q \rightarrow r \equiv p \rightarrow (q \rightarrow r)$  using the laws of logical equivalences.

Solution:

$$\begin{aligned}
 p \wedge q \rightarrow r &\equiv (p \wedge q) \rightarrow r && \text{(precedences of } \wedge \text{ and } \rightarrow \text{)} \\
 &\equiv \neg(p \wedge q) \vee r && \text{(equivalence } A \rightarrow B \equiv \neg A \vee B \text{)} \\
 &\equiv (\neg p \vee \neg q) \vee r && \text{(De Morgan's law)} \\
 &\equiv
 \end{aligned}$$

# Proving Equivalence Using Laws of Equivalences (1)

## Exercise

Show that  $p \wedge q \rightarrow r \equiv p \rightarrow (q \rightarrow r)$  using the laws of logical equivalences.

Solution:

$$\begin{aligned}
 p \wedge q \rightarrow r &\equiv (p \wedge q) \rightarrow r && \text{(precedences of } \wedge \text{ and } \rightarrow) \\
 &\equiv \neg(p \wedge q) \vee r && \text{(equivalence } A \rightarrow B \equiv \neg A \vee B) \\
 &\equiv (\neg p \vee \neg q) \vee r && \text{(De Morgan's law)} \\
 &\equiv \neg p \vee (\neg q \vee r) && \text{(associative law for } \vee) \\
 &\equiv
 \end{aligned}$$

# Proving Equivalence Using Laws of Equivalences (1)

## Exercise

Show that  $p \wedge q \rightarrow r \equiv p \rightarrow (q \rightarrow r)$  using the laws of logical equivalences.

Solution:

$$\begin{aligned}
 p \wedge q \rightarrow r &\equiv (p \wedge q) \rightarrow r && \text{(precedences of } \wedge \text{ and } \rightarrow) \\
 &\equiv \neg(p \wedge q) \vee r && \text{(equivalence } A \rightarrow B \equiv \neg A \vee B) \\
 &\equiv (\neg p \vee \neg q) \vee r && \text{(De Morgan's law)} \\
 &\equiv \neg p \vee (\neg q \vee r) && \text{(associative law for } \vee) \\
 &\equiv \neg p \vee (q \rightarrow r) && \text{(equivalence } \neg A \vee B \equiv A \rightarrow B) \\
 &\equiv
 \end{aligned}$$

# Proving Equivalence Using Laws of Equivalences (1)

## Exercise

Show that  $p \wedge q \rightarrow r \equiv p \rightarrow (q \rightarrow r)$  using the laws of logical equivalences.

Solution:

$$\begin{aligned}
 p \wedge q \rightarrow r &\equiv (p \wedge q) \rightarrow r && \text{(precedences of } \wedge \text{ and } \rightarrow \text{)} \\
 &\equiv \neg(p \wedge q) \vee r && \text{(equivalence } A \rightarrow B \equiv \neg A \vee B \text{)} \\
 &\equiv (\neg p \vee \neg q) \vee r && \text{(De Morgan's law)} \\
 &\equiv \neg p \vee (\neg q \vee r) && \text{(associative law for } \vee \text{)} \\
 &\equiv \neg p \vee (q \rightarrow r) && \text{(equivalence } \neg A \vee B \equiv A \rightarrow B \text{)} \\
 &\equiv p \rightarrow (q \rightarrow r) && \text{(equivalence } \neg A \vee B \equiv A \rightarrow B \text{)}
 \end{aligned}$$

Equivalence of formula  $A \rightarrow B \equiv \neg A \vee B$  can be proven easily using contradiction method of formula schema  $(A \rightarrow B) \leftrightarrow (\neg A \vee B)$ .

The equivalence  $p \wedge q \rightarrow r \equiv p \rightarrow (q \rightarrow r)$  is also known as **exportation law**.

# Proving Equivalence Using Laws of Equivalences (2)

## Exercise

Show that  $\neg(p \vee (\neg p \wedge q))$  and  $\neg p \wedge \neg q$  are equivalent using the laws of logical equivalences.

Solution:

# Proving Equivalence Using Laws of Equivalences (2)

## Exercise

Show that  $\neg(p \vee (\neg p \wedge q))$  and  $\neg p \wedge \neg q$  are equivalent using the laws of logical equivalences.

Solution:

$$\neg(p \vee (\neg p \wedge q)) \equiv$$

# Proving Equivalence Using Laws of Equivalences (2)

## Exercise

Show that  $\neg(p \vee (\neg p \wedge q))$  and  $\neg p \wedge \neg q$  are equivalent using the laws of logical equivalences.

Solution:

$$\begin{aligned} \neg(p \vee (\neg p \wedge q)) &\equiv \neg p \wedge \neg(\neg p \wedge q) && \text{(De Morgan's law)} \\ &\equiv \end{aligned}$$

# Proving Equivalence Using Laws of Equivalences (2)

## Exercise

Show that  $\neg(p \vee (\neg p \wedge q))$  and  $\neg p \wedge \neg q$  are equivalent using the laws of logical equivalences.

Solution:

$$\begin{aligned}
 \neg(p \vee (\neg p \wedge q)) &\equiv \neg p \wedge \neg(\neg p \wedge q) && \text{(De Morgan's law)} \\
 &\equiv \neg p \wedge (\neg(\neg p) \vee \neg q) && \text{(De Morgan's law)} \\
 &\equiv
 \end{aligned}$$

# Proving Equivalence Using Laws of Equivalences (2)

## Exercise

Show that  $\neg(p \vee (\neg p \wedge q))$  and  $\neg p \wedge \neg q$  are equivalent using the laws of logical equivalences.

Solution:

$$\begin{aligned}
 \neg(p \vee (\neg p \wedge q)) &\equiv \neg p \wedge \neg(\neg p \wedge q) && \text{(De Morgan's law)} \\
 &\equiv \neg p \wedge (\neg(\neg p) \vee \neg q) && \text{(De Morgan's law)} \\
 &\equiv \neg p \wedge (p \vee \neg q) && \text{(double negation law)} \\
 &\equiv && 
 \end{aligned}$$

# Proving Equivalence Using Laws of Equivalences (2)

## Exercise

Show that  $\neg(p \vee (\neg p \wedge q))$  and  $\neg p \wedge \neg q$  are equivalent using the laws of logical equivalences.

Solution:

$$\begin{aligned}
 \neg(p \vee (\neg p \wedge q)) &\equiv \neg p \wedge \neg(\neg p \wedge q) && \text{(De Morgan's law)} \\
 &\equiv \neg p \wedge (\neg(\neg p) \vee \neg q) && \text{(De Morgan's law)} \\
 &\equiv \neg p \wedge (p \vee \neg q) && \text{(double negation law)} \\
 &\equiv (\neg p \wedge p) \vee (\neg p \wedge \neg q) && \text{(distributive law)} \\
 &\equiv
 \end{aligned}$$

# Proving Equivalence Using Laws of Equivalences (2)

## Exercise

Show that  $\neg(p \vee (\neg p \wedge q))$  and  $\neg p \wedge \neg q$  are equivalent using the laws of logical equivalences.

Solution:

$$\begin{aligned}
 \neg(p \vee (\neg p \wedge q)) &\equiv \neg p \wedge \neg(\neg p \wedge q) && \text{(De Morgan's law)} \\
 &\equiv \neg p \wedge (\neg(\neg p) \vee \neg q) && \text{(De Morgan's law)} \\
 &\equiv \neg p \wedge (p \vee \neg q) && \text{(double negation law)} \\
 &\equiv (\neg p \wedge p) \vee (\neg p \wedge \neg q) && \text{(distributive law)} \\
 &\equiv \text{F} \vee (\neg p \wedge \neg q) && \text{(negation law)} \\
 &\equiv && 
 \end{aligned}$$

# Proving Equivalence Using Laws of Equivalences (2)

## Exercise

Show that  $\neg(p \vee (\neg p \wedge q))$  and  $\neg p \wedge \neg q$  are equivalent using the laws of logical equivalences.

Solution:

$$\begin{aligned}
 \neg(p \vee (\neg p \wedge q)) &\equiv \neg p \wedge \neg(\neg p \wedge q) && \text{(De Morgan's law)} \\
 &\equiv \neg p \wedge (\neg(\neg p) \vee \neg q) && \text{(De Morgan's law)} \\
 &\equiv \neg p \wedge (p \vee \neg q) && \text{(double negation law)} \\
 &\equiv (\neg p \wedge p) \vee (\neg p \wedge \neg q) && \text{(distributive law)} \\
 &\equiv \mathbf{F} \vee (\neg p \wedge \neg q) && \text{(negation law)} \\
 &\equiv (\neg p \wedge \neg q) \vee \mathbf{F} && \text{(commutative law)} \\
 &\equiv
 \end{aligned}$$

# Proving Equivalence Using Laws of Equivalences (2)

## Exercise

Show that  $\neg(p \vee (\neg p \wedge q))$  and  $\neg p \wedge \neg q$  are equivalent using the laws of logical equivalences.

Solution:

$$\begin{aligned}
 \neg(p \vee (\neg p \wedge q)) &\equiv \neg p \wedge \neg(\neg p \wedge q) && \text{(De Morgan's law)} \\
 &\equiv \neg p \wedge (\neg(\neg p) \vee \neg q) && \text{(De Morgan's law)} \\
 &\equiv \neg p \wedge (p \vee \neg q) && \text{(double negation law)} \\
 &\equiv (\neg p \wedge p) \vee (\neg p \wedge \neg q) && \text{(distributive law)} \\
 &\equiv \mathbf{F} \vee (\neg p \wedge \neg q) && \text{(negation law)} \\
 &\equiv (\neg p \wedge \neg q) \vee \mathbf{F} && \text{(commutative law)} \\
 &\equiv \neg p \wedge \neg q && \text{(identity law)}
 \end{aligned}$$

# Proving Validity Using Laws of Equivalences (1)

## Exercise

Show that  $(p \wedge q) \rightarrow (p \vee q)$  is valid using the laws of logical equivalences.

Solution:

# Proving Validity Using Laws of Equivalences (1)

## Exercise

Show that  $(p \wedge q) \rightarrow (p \vee q)$  is valid using the laws of logical equivalences.

Solution:

We must show that  $(p \wedge q) \rightarrow (p \vee q)$  is equivalent to  $\mathbb{T}$ .

$$(p \wedge q) \rightarrow (p \vee q) \equiv$$

# Proving Validity Using Laws of Equivalences (1)

## Exercise

Show that  $(p \wedge q) \rightarrow (p \vee q)$  is valid using the laws of logical equivalences.

Solution:

We must show that  $(p \wedge q) \rightarrow (p \vee q)$  is equivalent to  $\mathbb{T}$ .

$$\begin{aligned} (p \wedge q) \rightarrow (p \vee q) &\equiv \neg(p \wedge q) \vee (p \vee q) && \text{(equivalence } A \rightarrow B \equiv \neg A \vee B) \\ &\equiv \end{aligned}$$

# Proving Validity Using Laws of Equivalences (1)

## Exercise

Show that  $(p \wedge q) \rightarrow (p \vee q)$  is valid using the laws of logical equivalences.

Solution:

We must show that  $(p \wedge q) \rightarrow (p \vee q)$  is equivalent to  $\mathbf{T}$ .

$$\begin{aligned}
 (p \wedge q) \rightarrow (p \vee q) &\equiv \neg(p \wedge q) \vee (p \vee q) && \text{(equivalence } A \rightarrow B \equiv \neg A \vee B) \\
 &\equiv (\neg p \vee \neg q) \vee (p \vee q) && \text{(De Morgan's law)} \\
 &\equiv
 \end{aligned}$$

# Proving Validity Using Laws of Equivalences (1)

## Exercise

Show that  $(p \wedge q) \rightarrow (p \vee q)$  is valid using the laws of logical equivalences.

Solution:

We must show that  $(p \wedge q) \rightarrow (p \vee q)$  is equivalent to  $\mathbb{T}$ .

$$\begin{aligned}
 (p \wedge q) \rightarrow (p \vee q) &\equiv \neg(p \wedge q) \vee (p \vee q) && \text{(equivalence } A \rightarrow B \equiv \neg A \vee B) \\
 &\equiv (\neg p \vee \neg q) \vee (p \vee q) && \text{(De Morgan's law)} \\
 &\equiv (\neg p \vee p) \vee (\neg q \vee q) && \text{(commutative and} \\
 &&& \text{associative law for } \vee) \\
 &\equiv \mathbb{T}
 \end{aligned}$$

# Proving Validity Using Laws of Equivalences (1)

## Exercise

Show that  $(p \wedge q) \rightarrow (p \vee q)$  is valid using the laws of logical equivalences.

Solution:

We must show that  $(p \wedge q) \rightarrow (p \vee q)$  is equivalent to  $\mathbf{T}$ .

$$\begin{aligned}
 (p \wedge q) \rightarrow (p \vee q) &\equiv \neg(p \wedge q) \vee (p \vee q) && \text{(equivalence } A \rightarrow B \equiv \neg A \vee B) \\
 &\equiv (\neg p \vee \neg q) \vee (p \vee q) && \text{(De Morgan's law)} \\
 &\equiv (\neg p \vee p) \vee (\neg q \vee q) && \text{(commutative and} \\
 &&& \text{associative law for } \vee) \\
 &\equiv \mathbf{T} \vee \mathbf{T} && \text{(commutative and} \\
 &&& \text{negation law for } \vee) \\
 &\equiv && 
 \end{aligned}$$

# Proving Validity Using Laws of Equivalences (1)

## Exercise

Show that  $(p \wedge q) \rightarrow (p \vee q)$  is valid using the laws of logical equivalences.

Solution:

We must show that  $(p \wedge q) \rightarrow (p \vee q)$  is equivalent to  $\mathbf{T}$ .

$$\begin{aligned}
 (p \wedge q) \rightarrow (p \vee q) &\equiv \neg(p \wedge q) \vee (p \vee q) && \text{(equivalence } A \rightarrow B \equiv \neg A \vee B) \\
 &\equiv (\neg p \vee \neg q) \vee (p \vee q) && \text{(De Morgan's law)} \\
 &\equiv (\neg p \vee p) \vee (\neg q \vee q) && \text{(commutative and} \\
 &&& \text{associative law for } \vee) \\
 &\equiv \mathbf{T} \vee \mathbf{T} && \text{(commutative and} \\
 &&& \text{negation law for } \vee) \\
 &\equiv \mathbf{T} && \text{(domination law)}
 \end{aligned}$$

Note: from this result, we can write  $(p \wedge q) \Rightarrow (p \vee q)$ .

## Proving Validity Using Laws of Equivalences (2)

### Exercise

Show that  $(p \rightarrow q \wedge r) \rightarrow (p \rightarrow q)$  is valid using the laws of logical equivalences.

Solution:

## Proving Validity Using Laws of Equivalences (2)

### Exercise

Show that  $(p \rightarrow q \wedge r) \rightarrow (p \rightarrow q)$  is valid using the laws of logical equivalences.

Solution:

We must show that  $(p \rightarrow q \wedge r) \rightarrow (p \rightarrow q)$  is equivalent to  $\mathbb{T}$ .

$$\begin{aligned} & (p \rightarrow q \wedge r) \rightarrow (p \rightarrow q) \\ \equiv & \end{aligned}$$

## Proving Validity Using Laws of Equivalences (2)

### Exercise

Show that  $(p \rightarrow q \wedge r) \rightarrow (p \rightarrow q)$  is valid using the laws of logical equivalences.

Solution:

We must show that  $(p \rightarrow q \wedge r) \rightarrow (p \rightarrow q)$  is equivalent to  $\mathbb{T}$ .

$$\begin{aligned}
 & (p \rightarrow q \wedge r) \rightarrow (p \rightarrow q) \\
 \equiv & \neg(p \rightarrow q \wedge r) \vee (p \rightarrow q) && \text{(equivalence } A \rightarrow B \equiv \neg A \vee B) \\
 \equiv &
 \end{aligned}$$

# Proving Validity Using Laws of Equivalences (2)

## Exercise

Show that  $(p \rightarrow q \wedge r) \rightarrow (p \rightarrow q)$  is valid using the laws of logical equivalences.

Solution:

We must show that  $(p \rightarrow q \wedge r) \rightarrow (p \rightarrow q)$  is equivalent to  $\mathbb{T}$ .

$$\begin{aligned}
 & (p \rightarrow q \wedge r) \rightarrow (p \rightarrow q) \\
 \equiv & \quad \neg(p \rightarrow q \wedge r) \vee (p \rightarrow q) && \text{(equivalence } A \rightarrow B \equiv \neg A \vee B) \\
 \equiv & \quad \neg(\neg p \vee (q \wedge r)) \vee (\neg p \vee q) && \text{(equivalence } A \rightarrow B \equiv \neg A \vee B) \\
 \equiv &
 \end{aligned}$$

## Proving Validity Using Laws of Equivalences (2)

### Exercise

Show that  $(p \rightarrow q \wedge r) \rightarrow (p \rightarrow q)$  is valid using the laws of logical equivalences.

Solution:

We must show that  $(p \rightarrow q \wedge r) \rightarrow (p \rightarrow q)$  is equivalent to  $\mathbb{T}$ .

$$\begin{aligned}
 & (p \rightarrow q \wedge r) \rightarrow (p \rightarrow q) \\
 \equiv & \neg(p \rightarrow q \wedge r) \vee (p \rightarrow q) && \text{(equivalence } A \rightarrow B \equiv \neg A \vee B) \\
 \equiv & \neg(\neg p \vee (q \wedge r)) \vee (\neg p \vee q) && \text{(equivalence } A \rightarrow B \equiv \neg A \vee B) \\
 \equiv & (\neg(\neg p) \wedge \neg(q \wedge r)) \vee (\neg p \vee q) && \text{(De Morgan's law)} \\
 \equiv & 
 \end{aligned}$$

## Proving Validity Using Laws of Equivalences (2)

### Exercise

Show that  $(p \rightarrow q \wedge r) \rightarrow (p \rightarrow q)$  is valid using the laws of logical equivalences.

Solution:

We must show that  $(p \rightarrow q \wedge r) \rightarrow (p \rightarrow q)$  is equivalent to  $\top$ .

$$\begin{aligned}
 & (p \rightarrow q \wedge r) \rightarrow (p \rightarrow q) \\
 \equiv & \neg(p \rightarrow q \wedge r) \vee (p \rightarrow q) && \text{(equivalence } A \rightarrow B \equiv \neg A \vee B) \\
 \equiv & \neg(\neg p \vee (q \wedge r)) \vee (\neg p \vee q) && \text{(equivalence } A \rightarrow B \equiv \neg A \vee B) \\
 \equiv & (\neg(\neg p) \wedge \neg(q \wedge r)) \vee (\neg p \vee q) && \text{(De Morgan's law)} \\
 \equiv & (p \wedge (\neg q \vee \neg r)) \vee (\neg p \vee q) && \text{(De Morgan's law} \\
 & && \text{and double negation law)}
 \end{aligned}$$

$$\begin{aligned} &\equiv (p \wedge \neg q) \vee (p \wedge \neg r) \vee (\neg p \vee q) && \text{(distributive law)} \\ &\equiv \end{aligned}$$

$$\begin{aligned} &\equiv (p \wedge \neg q) \vee (p \wedge \neg r) \vee (\neg p \vee q) && \text{(distributive law)} \\ &\equiv (p \wedge \neg q) \vee (\neg p \vee q) \vee (p \wedge \neg r) && \text{(commutative law for } \vee \text{)} \\ &\equiv \end{aligned}$$

$$\begin{aligned} &\equiv (p \wedge \neg q) \vee (p \wedge \neg r) \vee (\neg p \vee q) && \text{(distributive law)} \\ &\equiv (p \wedge \neg q) \vee (\neg p \vee q) \vee (p \wedge \neg r) && \text{(commutative law for } \vee \text{)} \\ &\equiv (\neg\neg p \wedge \neg q) \vee (\neg p \vee q) \vee (p \wedge \neg r) && \text{(double negation law)} \\ &\equiv \end{aligned}$$

$$\begin{aligned}
&\equiv (p \wedge \neg q) \vee (p \wedge \neg r) \vee (\neg p \vee q) && \text{(distributive law)} \\
&\equiv (p \wedge \neg q) \vee (\neg p \vee q) \vee (p \wedge \neg r) && \text{(commutative law for } \vee \text{)} \\
&\equiv (\neg\neg p \wedge \neg q) \vee (\neg p \vee q) \vee (p \wedge \neg r) && \text{(double negation law)} \\
&\equiv \neg(\neg p \vee q) \vee (\neg p \vee q) \vee (p \wedge \neg r) && \text{(De Morgan's law)} \\
&\equiv
\end{aligned}$$

$\equiv$	$(p \wedge \neg q) \vee (p \wedge \neg r) \vee (\neg p \vee q)$	(distributive law)
$\equiv$	$(p \wedge \neg q) \vee (\neg p \vee q) \vee (p \wedge \neg r)$	(commutative law for $\vee$ )
$\equiv$	$(\neg\neg p \wedge \neg q) \vee (\neg p \vee q) \vee (p \wedge \neg r)$	(double negation law)
$\equiv$	$\neg(\neg p \vee q) \vee (\neg p \vee q) \vee (p \wedge \neg r)$	(De Morgan's law)
$\equiv$	$\mathbf{T} \vee (p \wedge \neg r)$	(negation law)
$\equiv$		

$\equiv$	$(p \wedge \neg q) \vee (p \wedge \neg r) \vee (\neg p \vee q)$	(distributive law)
$\equiv$	$(p \wedge \neg q) \vee (\neg p \vee q) \vee (p \wedge \neg r)$	(commutative law for $\vee$ )
$\equiv$	$(\neg\neg p \wedge \neg q) \vee (\neg p \vee q) \vee (p \wedge \neg r)$	(double negation law)
$\equiv$	$\neg(\neg p \vee q) \vee (\neg p \vee q) \vee (p \wedge \neg r)$	(De Morgan's law)
$\equiv$	$\mathbf{T} \vee (p \wedge \neg r)$	(negation law)
$\equiv$	$(p \wedge \neg r) \vee \mathbf{T}$	(commutative law for $\vee$ )
$\equiv$		

$\equiv$	$(p \wedge \neg q) \vee (p \wedge \neg r) \vee (\neg p \vee q)$	(distributive law)
$\equiv$	$(p \wedge \neg q) \vee (\neg p \vee q) \vee (p \wedge \neg r)$	(commutative law for $\vee$ )
$\equiv$	$(\neg\neg p \wedge \neg q) \vee (\neg p \vee q) \vee (p \wedge \neg r)$	(double negation law)
$\equiv$	$\neg(\neg p \vee q) \vee (\neg p \vee q) \vee (p \wedge \neg r)$	(De Morgan's law)
$\equiv$	$\mathbf{T} \vee (p \wedge \neg r)$	(negation law)
$\equiv$	$(p \wedge \neg r) \vee \mathbf{T}$	(commutative law for $\vee$ )
$\equiv$	$\mathbf{T}$	(domination law)

Note: from this result, we can write  $(p \rightarrow q \wedge r) \Rightarrow (p \rightarrow q)$ .