Propositional Logic 1: Motivation – Parse Tree

Mathematical Logic – First Term 2023-2024

MZI

School of Computing
Telkom University

SoC Tel-U

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Acknowledgements

This slide is compiled using the materials in the following sources:

1. *Discrete Mathematics and Its Applications* (Chapter 1), 8th Edition, 2019, by K. H. Rosen (primary reference).
2. *Discrete Mathematics with Applications* (Chapter 2), 5th Edition, 2018, by S. S. Epp.
3. *Logic in Computer Science: Modelling and Reasoning about Systems* (Chapter 1), 2nd Edition, 2004, by M. Huth and M. Ryan.
4. *Mathematical Logic for Computer Science* (Chapter 2, 3, 4), 2nd Edition, 2000, by M. Ben-Ari.
5. Discrete Mathematics 1 (2012) slides in Fasilkom UI by B. H. Widjaja.
6. Mathematical Logic slides in Telkom University by A. Rakhmatsyah and B. Purnama.

Some figures are excerpted from those sources. This slide is intended for internal academic purpose in SoC Telkom University. No slides are ever free from error nor incapable of being improved. Please convey your comments and corrections (if any) to `<pleasedontspam>@telkomuniversity.ac.id`.

Contents

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Can an information system with these specification be built? In other words, are these system specification consistent?

System specification consistency problem is one of the problems that can be solved using propositional logic which we are going to learn.

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1. *unary operator*; only requires one *operand*: negation ($\neg$ or $\sim$);
2. *binary operator*; requires two *operands*: conjunction ($\wedge$), disjunction ($\vee$), exclusive disjunction/ exclusive-or ($\oplus$), implication ($\rightarrow$), bi-implication ($\leftrightarrow$)

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Write in English the negation of following propositions:

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1. I didn't do nothing. (So, the speaker did something or not?)

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In Standard English, two negatives are understood to resolve to a positive; however, today as African American culture becomes more dispersed, the use of double negatives becomes increasingly prevalent and in many cases lead to confusions.

For examples:

1. I didn't do nothing. (So, the speaker did something or not?)
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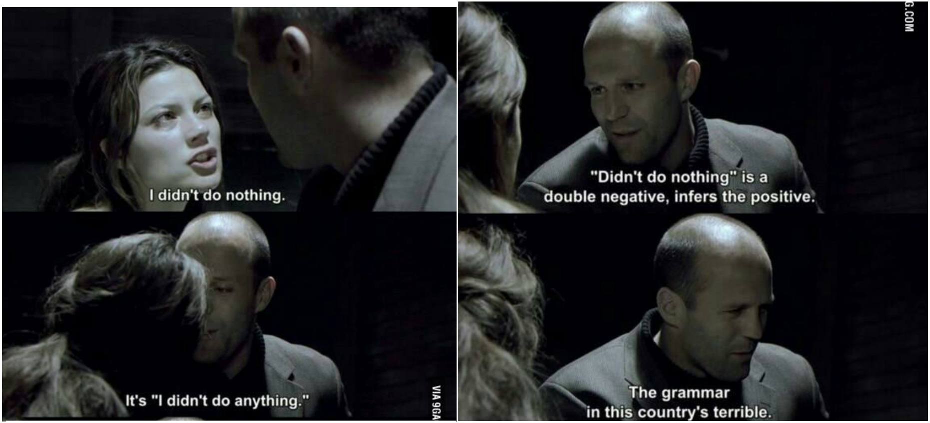
1. I didn't do nothing. (So, the speaker did something or not?)
2. I ain't got no money. (So, the speaker got some money or not?)
3. I'm not hungry no more. (So, the speaker is hungry or not?)


I didn't do nothing.
It's "I didn't do anything."
"Didn't do nothing" is a double negative, infers the positive.
The grammar in this country's terrible.

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Suppose we have following atomic propositions:

p : The sun rises in the east	$q : 2 \times 3 \leq 3^2$
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Write and determine the truth value of the following compound propositions: (1) $p \wedge \neg q$; (2) $\neg r \wedge \neg s$.

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Observe that $p \vee q$ is also true if both p and q are true.

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Example of Exclusive Disjunctions

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Suppose we have following atomic propositions:

p : Alex likes cookies q : Alex likes pizza
r : 2 is an even number s : 2 is a prime number

Write the compound proposition: (1) $p \oplus q$; (2) $r \oplus s$ and also determine the truth value of $r \oplus s$.

Solution:

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2. $r \oplus s$: "2 is an even number but it is not a prime number, or 2 is not an even number but it is a prime number"; or we can also write "2 is either an even number or a prime number, but not both". The truth of each of r and s

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2. $r \oplus s$: "2 is an even number but it is not a prime number, or 2 is not an even number but it is a prime number"; or we can also write "2 is either an even number or a prime number, but not both". The truth of each of r and s is true, therefore the truth of $r \oplus s$ is false.

Implication or Conditional Statement

Implication or Conditional Statement

Let p and q be propositions, then $p \rightarrow q$ is also a proposition which is called an implication or a conditional statement of the proposition "if p, then q". In an implication $p \rightarrow q$, p is called the hypothesis/ antecedent/ premise and q is called the conclusion/ consequence.

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if p then q	q if p
p implies q	q is implied by p
p is <u>sufficient</u> for q	q is <u>necessary</u> for p

$p \rightarrow q$ is false (F) if p is true but q is false, **otherwise** $p \rightarrow q$ **is** true

Truth table for conditional statement $p \rightarrow q$

p	q	$p \rightarrow q$
T	T	

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T	F	

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T	T	T
T	F	F
F	T	

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T	T	T
T	F	F
F	T	T
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p	q	$p \rightarrow q$
T	T	T
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Examples of Conditional Statements

Example

Consider following propositions:

p : "Alex's final exam is 100"

q : "Alex's final grade is A"

$p \rightarrow q$:

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$p \rightarrow q$ is false (F) when Alex's final exam is 100,

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$p \rightarrow q$ is true (T) when:

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$p \rightarrow q$ is true (T) when:

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Examples of Conditional Statements

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$p \rightarrow q$ is true (T) when:

- Alex's final exam is not 100
- Alex's final grade is A.

Exercise

At the beginning of a season, an owner of a basketball club once said, "If my club win this season's final, then I will sell this club". When the season ended, the club was relegated to the lower division, yet the owner still sold his club. Does the act of selling the club violate the owner's previous statement?

Contrapositive, Converse, and Inverse

Let $p \longrightarrow q$ be an implication, then:

- the **contrapositive** (sometimes called contraposition) of $p \longrightarrow q$ is $\neg q \longrightarrow \neg p$
- the **converse** of $p \longrightarrow q$ is $q \longrightarrow p$
- the **inverse** of $p \longrightarrow q$ is $\neg p \longrightarrow \neg q$

The truth table for contrapositive, converse, and inverse of an implication:

					contrapositive	converse	inverse
p	$\neg p$	q	$\neg q$	$p \rightarrow q$	$\neg q \rightarrow \neg p$	$q \rightarrow p$	$\neg p \rightarrow \neg q$
T	F	T	F				

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T	F	F	T	F	F	T	T
F	T	T	F	T	T	F	F
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Observe that the truth of $p \rightarrow q$ is identical to the truth of

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F	T	T	F	T	T	F	F
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Observe that the truth of $p \rightarrow q$ is identical to the truth of $\neg q \rightarrow \neg p$, the truth of $q \rightarrow p$ is identical to the truth of $\neg p \rightarrow \neg q$. In this condition, we say $p \rightarrow q$ is equivalent to $\neg q \rightarrow \neg p$ and

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Observe that the truth of $p \to q$ is identical to the truth of $\neg q \to \neg p$, the truth of $q \to p$ is identical to the truth of $\neg p \to \neg q$. In this condition, we say $p \to q$ is equivalent to $\neg q \to \neg p$ and $q \to p$ is equivalent to $\neg p \to \neg q$.

Remark

Every conditional statement is equivalent to its contrapositive.

Bi-implication or Biconditional Statement

Bi-implication

Let p and q be propositions, then $p \leftrightarrow q$ is also a proposition which is called a bi-implication or a biconditional statement of the propositions "p if and only if q".

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p if and only if q	p iff q, (*iff* is a shorthand for if and only if)
if p then q, and conversely	p is necessary and sufficient for q
p is equivalent to q	p and q are equivalent

$p \leftrightarrow q$ is true (T) **precisely when** p and q have **the same truth values,** and is false otherwise.

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Truth table for a biconditional statement:

p	q	$p \leftrightarrow q$
T	T	

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Truth table for a biconditional statement:

p	q	$p \leftrightarrow q$
T	T	T
T	F	

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Truth table for a biconditional statement:

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Example of Biconditional Statement

Example

Consider following propositions:

p : "Alex's final grade was not less than 50"

q : "Alex passed the class"

$p \leftrightarrow q$:

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Consider following propositions:

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$p \leftrightarrow q$: "Alex's final grade was not less than 50 **if and only if** he passed the class"

$p \leftrightarrow q$

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1. Alex's final grade was not less than 50 **and** he passed the class; or

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$p \leftrightarrow q$ is true (T) when

1. Alex's final grade was not less than 50 **and** he passed the class; or
2. Alex's final grade was less than 50 **and** he didn't pass the class.

$p \leftrightarrow q$ is false (F) when

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Example of Biconditional Statement

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Consider following propositions:
p : "Alex's final grade was not less than 50"
q : "Alex passed the class"
$p \leftrightarrow q$: "Alex's final grade was not less than 50 **if and only if** he passed the class"
$p \leftrightarrow q$ is true (T) when

1. Alex's final grade was not less than 50 **and** he passed the class; or
2. Alex's final grade was less than 50 **and** he didn't pass the class.

$p \leftrightarrow q$ is false (F) when

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Contents

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As in arithmetic expressions, we use parentheses "(" and ")" to specify or clarify the order of an operation in a compound proposition.

Exercise

Put the parentheses to clarify the precedences of logical operators in the following compound propositions:

1. $p \vee q \wedge r$
2. $\neg p \vee q$
3. $p \wedge q \rightarrow r$
4. $p \rightarrow \neg q \wedge r$
5. $\neg p \vee q \rightarrow r \wedge \neg s$

Solution:

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Contents

Propositional Formulas

Propositional Formulas

A propositional formula may consist of:

1. propositional constant: T (true) or F (false)
2. atomic variables:

$$p, p_1, p_2, \ldots$$
$$q, q_1, q_2, \ldots$$
$$r, r_1, r_2, \ldots$$

3. propositional operators: $\neg, \wedge, \vee, \oplus, \rightarrow, \leftrightarrow$

and is constructed using following rules:

1. every atomic proposition is a propositional formula
2. if A and B are propositional formulas, then $\neg A$, $A \wedge B$, $A \vee B$, $A \oplus B$, $A \rightarrow B$, $A \leftrightarrow B$, are all propositional formulas.

Example of Propositional Formulas

Example

According to the definition of propositional formulas, we infer that

1. $p \wedge q$ is a propositional formula
2. $pq\vee$ is not a propositional formula
3. $\neg\neg\left(\neg p \to \neg\neg r\right)$

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5. $p \vee q\vee \to r \oplus s$ is not a propositional formula
6. $p \oplus p \vee q \to r \wedge s$ is a propositional formula that can be rewritten as $(p \oplus (p \vee q)) \to (r \wedge s)$
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8. $\neg((p \to q)\neg(r \oplus s))$ is not a propositional formula

Subformula

Subformula

1. A formula A is a subformula of A itself.
2. If A and B are two propositional formulas used to construct a more complex propositional formula C, then A and B are proper subformulas of C.
3. Subformula is transitive: if A is a subformula of B and B is a subformula of C, then A is a subformula of C.

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let A be a formula $(p \wedge q) \rightarrow (r \vee s)$, then the subformulas of A are: (1)

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let A be a formula $(p \wedge q) \to (r \vee s)$, then the subformulas of A are: (1) $(p \wedge q) \to (r \vee s)$, (2) $p \wedge q$, (3) $r \vee s$, (4)

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List all subformulas of $(p \to q) \vee (q \to p)$

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- $\neg r$
- p
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Exercise

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- $(\neg p \wedge q)$
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- $\neg r$
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Parse Tree

Parse tree can be used to visualize the structure of a propositional formula. For instance, the parse tree for $(\neg p \wedge q) \rightarrow (p \wedge (q \vee \neg r))$ is

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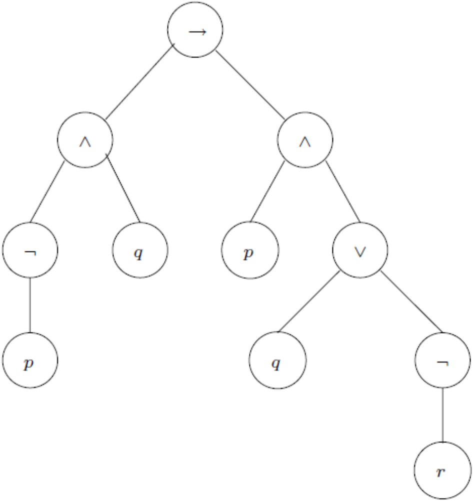

Exercise

Draw the parse tree of each of these formulas:

1. $\neg(p \vee (q \rightarrow \neg p)) \wedge r$
2. $(\neg p \wedge q) \rightarrow (p \wedge (q \vee \neg r))$
3. $\neg((q \rightarrow \neg p) \wedge (p \rightarrow r \vee q))$